Laser Cavity Analysis

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A Scanning Fabry-Perot spectrum analyzer with a free spectral range of 7.5GHz was used to examine the laser cavity of an Argon laser. By examining the separation in frequency between consecutive transmitted frequencies of the laser, the length of the laser cavity was determined to be (0.454±0.045) m. From the interference pattern created by the Doppler broadened spectrum of the laser the temperature of the laser, cavity was estimated to be (1400±200) K.

INTRODUCTION

A laser cavity doesn't usually emit a single frequency of light. Instead it produces a small spectrum of frequencies. An interference pattern of the frequencies within this spectrum can be created using a spectrum analyzer. By measuring the separations between consecutive constructive interferences in the interference pattern the length of the laser cavity can be calculated. Therefore it would not be possible to use this method to determine the length of the laser cavity if the laser did not produce a spectrum of frequencies. The most predominant reason why a laser produces a small spectrum of frequencies instead of just one is the Doppler effect.

A Doppler effect occurs when an object emitting a wave moves towards or away from an observer. The frequency of the emitted wave is shifted relative to the motion of the object. The Doppler effect can be applied to many physical systems. A train's whistle is a classic example of the Doppler effect. When a train approaches an observer, the frequency of its whistle will increase and when it moves away from the observer the frequency of its whistle will decrease. The Doppler effect can also be applied to moving atoms within a laser cavity. Because the atoms within a laser cavity are all moving with different speeds the laser will output a spectrum of light. This is called a Doppler broadening of light and because it occurs the temperature of the laser cavity can be estimated. In gas lasers[1] emitting in the visible part of the spectrum the predominant broadening mechanism is the Doppler effect.

THEORY

In this experiment the spectrum broadening is predominantly due to the Doppler effect because a gaseous Argon laser emitting blue light was used (Figure 1).

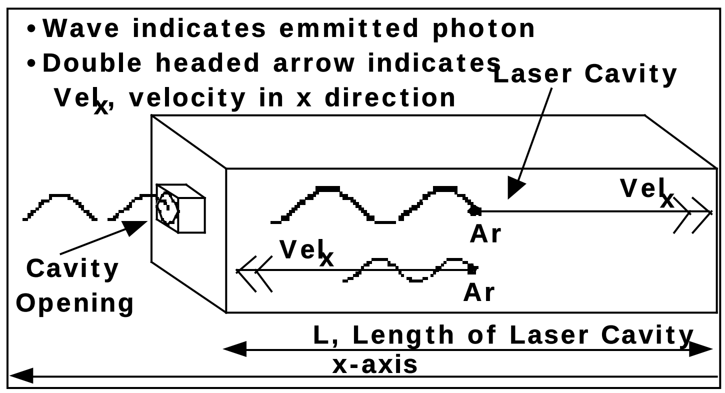

Figure 1. Inside the laser cavity, the photons produced by the Argon atoms are affected by the atom's motion.

The photons emitted are produced by the Argon atoms inside the laser cavity. When stationary, an Argon atom emits a photon of frequency ν_o. However within the laser cavity the Argon atoms are in motion. The component of their velocity along the x-axis, Vel_x, will affect the frequency of the laser light emitted. Knowing that the photons[2] travel at the speed of light, c, the emitted frequency, ν_e, is

$$\nu_e = \nu_o\left(1 + \frac{Vel_x}{c}\right) \qquad (1).$$

Simplifying equation (1), the atom's velocity in the x direction can be solved for as

$$Vel_x = c\frac{\nu_e - \nu_o}{\nu_o} \qquad (2).$$

Many frequencies can be produced due to the Doppler effect. But the laser emits only a few of these frequencies. Those frequencies that are emitted are related to the length of the laser cavity. The length of the laser cavity, L, will reinforce waves with wavelengths of length

$$\lambda = \frac{2L}{n} \qquad \text{where } n=1,2,3,... \qquad (3).$$

These waves and the length of the laser cavity will damp out waves not fitting equation (3). The frequency of a wave in terms of its wavelength is

$$\nu = \frac{Vel}{\lambda} \qquad (4).$$

Combining equations (3) and (4) and knowing that photons travel at the speed of light, c, the possible emitted frequencies from the laser cavity are

$$\nu = \frac{nc}{2L} \qquad \text{where } n=1,2,3,... \qquad (5).$$

Using equation (5) the separation in frequency between two consecutive emitted frequencies, $\Delta\nu_n$, can be calculated to be

$$\Delta\nu_n = \nu_{n+1} - \nu_n = \frac{c}{2L} \qquad (6).$$

The light emitted from the laser is then sent into a spectrum analyzer where it is separated into the different frequencies of which it is composed. The different frequencies constructively and destructively interfere with each other creating an interference pattern (Figure 2).

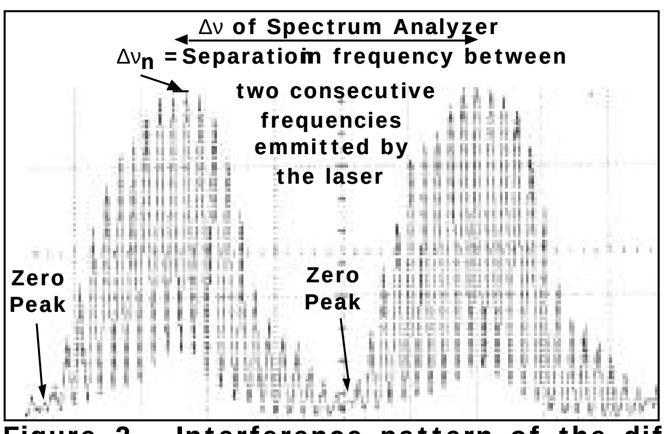

Figure 2. Interference pattern of the different frequencies emitted from the laser cavity.

In the interference pattern each small peak represents a frequency produced by the laser. The spectrum produced by the laser is represented by the group of small peaks which combine to form a wide peak. The wide peak, the emitted spectrum, represents the Doppler broadening of the laser beam and can be fit by a Gaussian distribution.

The Doppler broadening of the laser beam can be used to experimentally calculate the temperature of the laser cavity. The theory assumes[3] that the Argon atoms' velocities are constant and their vibrations are undisturbed during emission. Because the laser cavity is in thermal equilibrium,[2] the velocities of the Argon atoms in the x direction can be described by a Maxwellian probability distribution function

$$P(Vel_x) = \sqrt{\frac{M}{2\pi kT}}\, e^{\left(-\frac{MVel_x^2}{2kT}\right)} \qquad (7).$$

Applying equation (1) for velocity in the x direction equation (7) becomes

$$P(Vel_x) = \sqrt{\frac{M}{2\pi kT}}\, e^{\left(-\frac{Mc^2}{2kT}\frac{(\nu_e - \nu_o)^2}{\nu_o^2}\right)} \qquad (8).$$

A Gaussian fit of the Doppler broadened spectrum depicted in Figure 2 will be of the form

$$G(\nu_e) = I_o e^{\left(-\frac{(\nu_e - \nu_o)^2}{\sigma^2}\right)} \qquad (9).$$

By equating the exponents of equations (8) and (9), the temperature of the laser cavity can be solved for

$$T = \frac{Mc^2\sigma^2}{2k\nu_o^2} \qquad (10).$$

SETUP/PROCEDURE

A COHERENT (model 240-1-A) spectrum analyzer is attached to the spectrum analyzer control unit. The analyzer is a Fabry-Perot interferometer whose mirrors can adjust the wedge angle using a piezoelectric crystal attached to an externally applied ramp voltage. The output of the spectrum analyzer is observed on a digital oscilloscope (HP 54600B).

Adjust the height and position of the spectrum analyzer so that the laser beam strikes its center. The oscilloscope should now be displaying modes. If the laser beam reflects directly back into the laser cavity an interference will occur. This reflection can be moved away from the laser opening by adjusting the lens angle knobs. The position of the spectrum analyzer should be adjusted until the modes on the oscilloscope appear to be the largest and most defined.

DATA AND ANALYSIS

By definition the horizontal axis of the oscilloscope is in units of time. However, for the data to be useful a conversion factor from time to frequency must be estimated. The free spectral range of a spectrum analyzer is a constant value which corresponds to the range of frequencies that can be swept through without repetition. The free spectral range of a spectrum analyzer is related to the separation between its mirrors. For this spectrum analyzer the free spectral range is known to be 7.5 GHz[4]. Since the oscilloscope and spectrum analyzer control unit have been adjusted so that the spectrum repeats itself the conversion factor, γ, from ms to GHz can be experimentally measured.

The experimental value for the time between repeated frequencies is (9.127±0.047)ms. The conversion factor, γ, from ms to GHz is

$$\gamma = \frac{7.5GHz}{9.127ms} = (0.822 \pm 0.004)\frac{GHz}{ms} \quad (11).$$

The measured values for the left spectrum of peak frequencies in Figure 2 are displayed in Table 1.

Table 1. Data for the peak frequencies within the left envelope in Figure 2. Δt represents the difference in time between each peak and the previous peak.

Peak #	'Time (ms)'	'Frequency (Hz)'	'Voltage (V)'	'Δt (ms)'
0	18.68	1.54e+10	0.06719	0.40
1	18.28	1.50e+10	0.07812	0.44
2	17.84	1.47e+10	0.09062	0.36
3	17.48	1.44e+10	0.1038	0.44
4	17.04	1.40e+10	0.1134	0.40
5	16.64	1.37e+10	0.1206	0.40
6	16.24	1.33e+10	0.1244	0.40
7	15.84	1.30e+10	0.1284	0.44
8	15.4	1.27e+10	0.1334	0.40
9	15	1.23e+10	0.135	0.40
10	14.6	1.20e+10	0.1347	0.40
11	14.2	1.17e+10	0.1334	0.40
12	13.8	1.13e+10	0.1288	0.40
13	13.4	1.10e+10	0.1194	0.40
14	13	1.07e+10	0.1069	0.40
15	12.6	1.04e+10	0.09031	0.40
16	12.2	1.00e+10	0.08219	0.40
17	11.8	9.70e+09	0.07875	0.40
18	11.4	9.37e+09	0.07688	0.44
19	10.96	9.01e+09	0.07156	0.36
20	10.6	8.71e+09	0.06781	0.40
21	10.2	8.38e+09	0.065	0.00

From Table 1 the experimental value for Δt is (0.40±0.04) ms. Because Δt calculates the time between two consecutive peaks it can be used to calculate the experimental value for $\Delta\nu_n$

$$\Delta\nu_n = \Delta t \gamma = (0.33 \pm 0.033)GHz \quad (12).$$

Using this experimental value and equation (6) an experimental value for the length of the laser cavity can be determined

$$L = \frac{c}{2\Delta\nu_n} = (0.454 \pm 0.045)m \quad (13).$$

Using the data from Table 1 the temperature of the laser cavity can also be estimated. Figure 3 is a graph of the peak data from Table 1.

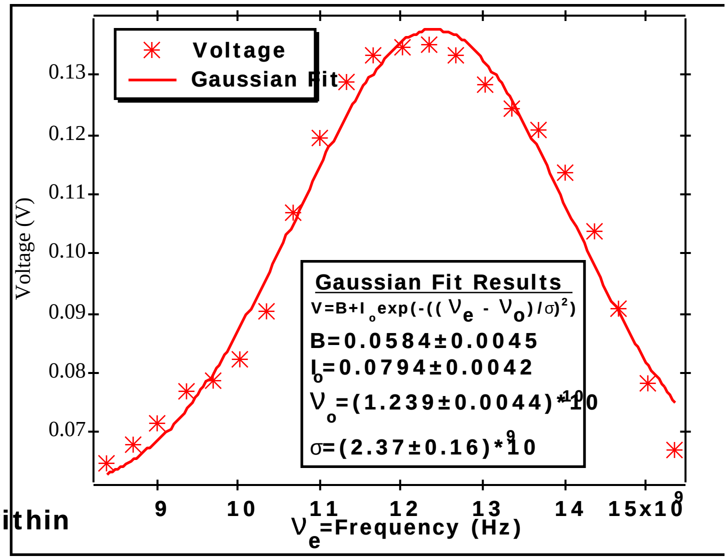

Figure 3. Graph of the peak data from Table 1 with a Gaussian fit.

In Figure 3, "ν_o" is the frequency where the peak occurs but is not the correct value for the frequency of the photons emitted from the Argon atoms when at rest. The value of "ν_o" in Figure 3 is related to the arbitrary offset of the time scale on the oscilloscope. The average wavelength[2] of a blue Argon laser beam is 488.0 nm. Using this and equation (4) the value for the average frequency, ν_o, of a blue Argon laser beam is $6.14*10^{14}$ Hz. Using the value for σ from the Gaussian fit in Figure 3 and equation (10) an experimental value for the temperature of the laser cavity can be determined

$$T = \frac{(2.98*10^{-26}kg)c^2(2.37*10^9 Hz)^2}{2k_B(6.14*10^{14}Hz)^2}$$
$$= (1400 \pm 200)K \quad (14).$$

CONCLUSION

This year there have been a wide variety of results from performing this experiment, which may be due to the fact that many small details affect its outcome. One detail I noticed, that may have previously been overlooked, was that the measurement of the oscilloscope can be affected by its proximity to the spectrum analyzer control unit. The spectrum analyzer control unit adjusts the spectrum analyzer using a ramp voltage. Although the coaxial cables connected to the spectrum analyzer control unit are very well sheathed to prevent interference the spectrum analyzer control unit itself seemed be creating the interference. So I moved the oscilloscope as far away as possible from the spectrum analyzer control unit when collecting data. Another small detail which could be overlooked is related to the Gaussian function in equation (9). The denominator in the exponent of the typical Gaussian function is 2σ. However, the denominator in the exponent of the Gaussian fit that Igor applies is just σ, a difference of a factor of 2.

I feel that the theory discussed here describes the phenomenon correctly. Because the actual length of the laser is approximately 0.48m long the experimental laser cavity value of (0.454±0.045) m seems very likely. The experimental values for the temperature of the laser cavity this year have ranged from nearly 0 K to 4,000 K. Both extremes seem to be impossible values for the temperature of the laser cavity. Although I was not able to find any published experimental values for the temperature of the laser cavity I feel confident that my experimental value of (1400±200) K is correct

[1]Donald C. O'Shea, Introduction to Lasers and Their Applications, (Addison-Wesley Publishing Company, California, 1978) pp. 79-89, 284-286.

[2]William T. Silfvast, Laser Fundamentals, (Cambridge University Press, USA, 1996) pp. 100-111

[3]John M. Stone, Radiation and Optics, (McGraw-Hill Book Company, New York, 1963) pp. 261-265.

[4]Coherent, Spectrum Analyzer Users Manuel p. 5.