Building a Passive Robot for Active Learning

A Junior Independent Study Self-Design

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(Dated: May 5, 2017)

We found that creating a passive walking robot out of plastic Tinkertoys is possible. This toy is able to walk down a slope without a motor, simply by the force of gravity. To understand this system, we videotaped multiple runs of the walker taking multiple steps down an incline. Then we used motion tracking and video-editing software to measure the length in time and distance of each step. By plotting these measurements against the Cornell group that inspired this project's average measurements, we discovered that the differences in the mass distribution for a wooden and plastic Tinkertoy walker change the behavior of the system. We found that the time step was faster in our system (0.34 ± 0.04 s compared to 0.47 s) and the step distance was longer in our system (1.8 ± 0.3 cm compared to 1.3 cm). The percent difference between our group and the Cornell group for both of these values was 32%. These constrained double pendulum systems are extremely sensitive to the building parameters and the initial conditions. Because of this, this system is hard to accurately predict using simplified models.

I. INTRODUCTION

The field of robotics is hot right now across the STEM world. The applications are seemingly endless. When people think of robotics they usually think of metallic, motor-driven machines. That does an injustice to the expansive possibilities of robots, including soft and passive robotics.

Passive robotics is an important subfield for multiple reasons. If we learn how to efficiently use energy that is already a part of a system we can create robots that are more energy efficient. For our study passive robotics are significant because they teach us why certain movements are efficient and prevalent in our world.

In our work we looked at passive walkers, which are walkers that are able to move down a slope simply by the force of gravity driving them down it. This kind of observed movement teaches us why humans have a tendency walk on earth. Since the simple walkers we model and create are able to move down a slope without a driving force it means that walking is extremely efficient in earth's gravity field. This reason is why humans are able to walk for such long distances without getting tired. Other gaits, such as running or hopping, are less efficient under earth's gravity because they require more inputted energy.

We hope that through our work we can create an approachable way for children to understand the complex mechanics behind the efficiency of these moments. Our goal is to replicate a Tinkertoy model of the simple walker originally created by a group at Cornell University [2].

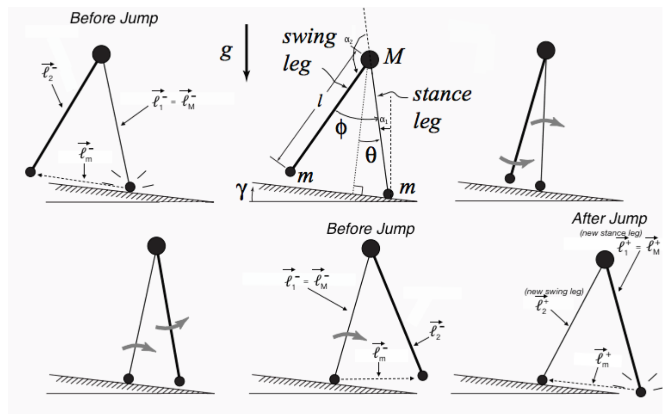

FIG. 1. Diagram of our passive walker simulation's variables. This diagram helps clarify the meaning of key terms, such as swing leg and stance leg. It also denotes which angle is which in our theory. The picture specifies what vectors and angles correspond with walker before and after the jump conditions are applied. The first five parts of the diagram represent the movement of a complete step before a jump, while the last part shows the most directly after the jump occurs, when the walker puts the swing foot down and Equation 10 is satisfied. We modified a diagram from Mariano Garcia's Thesis to have the specifications of our simulation. [4]. For our work we included angles to simplify our solution (i.e. the alpha angles) and to allow more variation, specifically the dependence on β, g, and ℓ variables were not work out of the mathematics.

II. THEORY

The first goal of our Clare Boothe Luce work was to solve for the equations of motion and the jump conditions of a passive two-dimensional walker. First, we set up the angles of the walker and the vectors that we would use to define the position of each of the key three spots - the two feet masses m and the hip mass M - as shown in Figure 1. These specifications allow use to define the constraints of the system as

$$\vec{\ell_1} = \ell\{-\sin(\alpha_1(t)), \cos(\alpha_1(t)), 0\} \tag{1}$$

$$\vec{\ell_2} = \ell\{-\sin(\alpha_1(t)+\alpha_2(t)), \cos(\alpha_1(t)+\alpha_2(t)), 0\} \tag{2}$$

$$\vec{\ell_M} = \vec{\ell_1} \tag{3}$$

$$\vec{\ell_m} = \vec{\ell_1} + \vec{\ell_2}, \tag{4}$$

where ℓ is the length of each leg and the α angles and the vectors are as defined in the Figure 1.

From there we solved for the Lagrangian $L = T - V$ of the constrained double pendulum system, where we define the kinetic energy T to be

$$T = \frac{\ell^2}{2}((2m + M + 2m\cos(\alpha_2(t)))\alpha_1{}'(t)^2 + 4m\cos^2(\frac{\alpha_2(t)}{2})\alpha_1{}'(t)\alpha_2{}'(t) + m\alpha_2{}'(t)^2)) \tag{5}$$

and the potential energy V to be

$$V = g\ell((m+M)\cos(\alpha_1(t)) + m\cos(\alpha_1(t)+\alpha_2(t))), \tag{6}$$

and got

$$L = \frac{1}{2}\ell(-2g((m+M)\cos(\alpha_1(t)) + m\cos(\alpha_1(t)+\alpha_2(t))) + \ell\alpha_1{}'(t)^2(2m\cos(\alpha_2(t)) + 2m + M) + 4m\ell\alpha_1{}'(t)\alpha_2{}'(t)\cos^2(\frac{\alpha_2(t)}{2}) + m\ell\alpha_2{}'(t)^2) \quad [12]. \tag{7}$$

From there we used the Euler-Lagrange equations to solve for the equations of motion. The first equation was

$$\ddot{\theta} = \frac{1}{\ell(\beta\cos^2(\phi) - \beta - 1)}(g((\beta+1)\sin(\gamma-\theta) - \beta\cos(\phi)\sin(\phi+\gamma-\theta))\beta\ell\dot{\phi}^2\sin(\phi) + 2\beta\ell\dot{\phi}\dot{\theta}\sin(\phi) + \beta\ell\dot{\theta}^2\sin(\phi)(\cos(\phi)-1)), \tag{8}$$

where $\beta = m/M$ and γ is the slope angle. The other equation was

$$\ddot{\phi} = \frac{1}{\ell(\beta\cos^2(\phi) - \beta - 1)}(g(2(\beta+1)\sin^2(\frac{\phi}{2})\sin(\gamma-\theta) + (\beta(-\cos(\phi)) + \beta + 1)\sin(\phi+\gamma-\theta)) + \beta\ell\dot{\phi}^2\sin(\phi)(\cos(\phi)-1) + 4\beta\ell\dot{\phi}\dot{\theta}\sin^2(\frac{\phi}{2})\sin(\phi) + \ell\dot{\theta}^2\sin(\phi)(2\beta\cos(\phi) - 2\beta - 1)) \quad [12]. \tag{9}$$

Originally, we defined our terms with α_1 and α_2 because it was the simplest approach for us to figure out the constraints on the system. We converted these to θ and ϕ for our equations of motion, using $\alpha_1 = \theta - \gamma$ and $\alpha_2 = -\phi + \pi$, because the Cornell group we were comparing our research to used these angles instead. Also, we made this change because the jump condition is best expressed in terms of θ and ϕ, as shown in Equation 10.

With the equations of motion we could plot only one stride because in order to have a continuous walking motion there has to be moments where the stance and swing leg switch and they become the other. This moment is where the jump conditions occur in our simulation. The approximate moment of this jump is when

$$\phi(t) - 2\theta(t) = 0. \quad [4] \tag{10}$$

This is because the swing leg has reached a stretched out point where the weight of the body starts to lean on it. The jump conditions that we found were

$$\begin{pmatrix} \theta^+(t) \to -\theta^-(t) \\ \theta^{+\prime}(t) \to -\frac{2\theta^{-\prime}(t)\cos(\phi^-(t))}{-\beta+\beta\cos(4\theta^-(t))-2} \\ \phi^+(t) \to -2\theta^-(t) \\ \phi^{+\prime}(t) \to -\frac{4\theta^{-\prime}(t)\sin^2(\theta^-(t))\cos(\phi^-(t))}{-\beta+\beta\cos(4\theta^-(t))-2} \end{pmatrix} \tag{11}$$

where the pluses denote after the jump conditions have been applied and the minuses denote the moment before they have been applied [12]. These jump conditions and equations were then applied to an Objective-C code, as shown in the Appendix.

In Mariano Garcia's 1998 Thesis, they expanded the equations of motion and jump conditions into three dimensions [4]. These mathematics are much more complicated and sensitive than those that we investigated in the two-dimensional model. The theoretical model of a three-dimensional walker is shown in Figure 2. The most important part about this three-dimensional model is that in the Cornell group's paper "An uncontrolled walking toy," they mention that the simplified theoretical model does not predict the actual motion of the Tinkertoy model [2].

III. PROCEDURE

The setup of our experiment, shown in Figures 3 and 4, is straightforward and discussed in detail in multiple papers written by the robotics group at Cornell [2]. We used plastic Tinkertoy pieces to create the toy model shown in Figure 4, as opposed to the Cornell group that had wooden pieces available to them, as shown in Figure 3. Additionally, we created a sloped platform for the robot to walk down according to the specifications from the paper [2]. One difference in our experiment from the original was that we used a plank of wood and a plastic cylinder to prop it up to create our slope. Using the plastic cylinder we were able to easily manipulate the angle if we wished to. After successfully creating this toy as close to the original measurements as possible, we

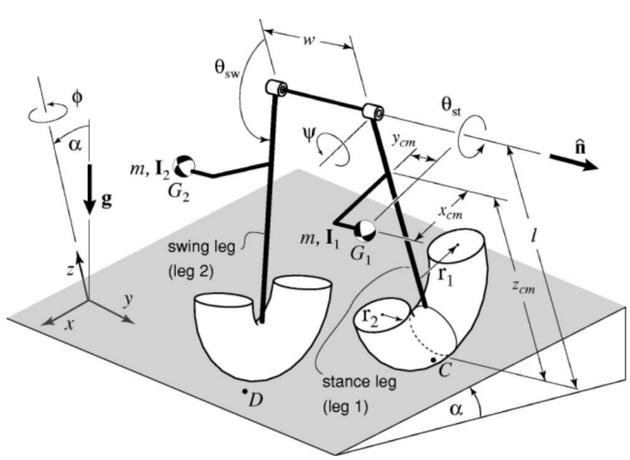

FIG. 2. Theoretical setup of a passive walker. This diagram is from the Cornell group's paper "Prediction of stable walking for a toy that cannot stand" and it connects the dimensions discussed in the Cornell group's theory and computational papers to their physical toy model [1]. This group eventually discovered that their simple simulations did not accurately predict the behavior of their passive robot.

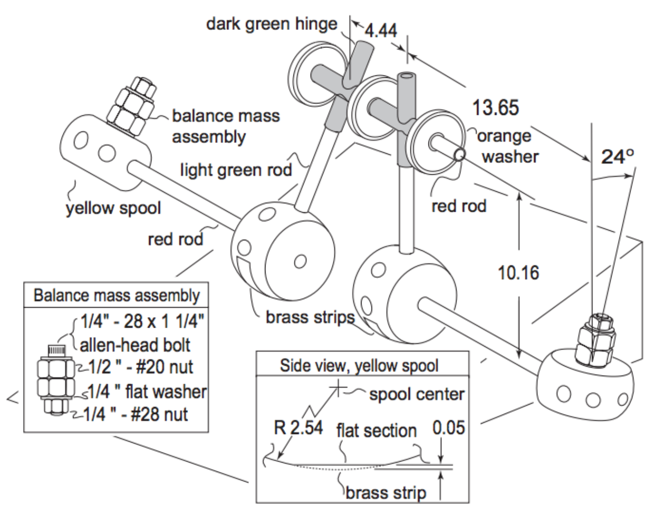

FIG. 3. Diagram of the Tinkertoy passive walking robot created by the Cornell group. This picture specifies the measurements of each part necessary to create a stable passive walking robot. We used this as a start-off point to create our initial robot. [2]. The overall mass of their walker was approximately 120 g with average foot balances of 50 g.

measured the step sizes and time lengths using a motion tracking program.

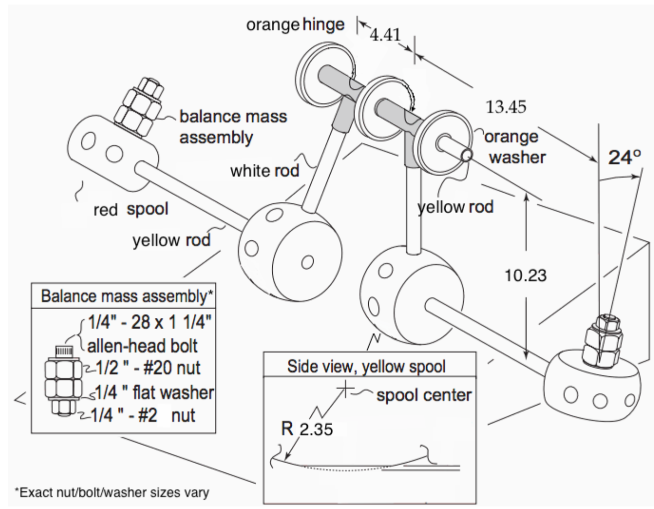

FIG. 4. Diagram of the Tinkertoy passive walking robot created by the Wooster group. This picture specifies the measurements of each part of our passive walking robot. We used the Cornell group's diagram as a base for our own and only a measurements varied [2]. The overall mass of our walker was 145.79 ± 0.03 g with the average of the masses on the foot balances being 41.08 ± 0.02 g.

IV. RESULTS

Creating our passive robot required tinkering with the set that we had. We were using the specs for a robot that had been built when wooden Tinkertoys were readily available. In today's market, wooden Tinkertoys are collectables and the mass produced toys are plastic. Also, the pieces that came in our set are slightly different sizes than the ones in the wooden sets, as you can tell by comparing the measurements on Figures 3 and 4. The orange pieces had to be hollowed out and the spacing for most of the parts had to be altered, though the overall look is the same. One of the big differences is the mass distribution. Of the 120 g of the Cornell toy, approximately 100 g was the masses add to the foot balances [2]. In our toy, though, of the 146 g approximately 82 g was the foot masses.

We then compared our results for the step time and step distance to the original Tinkertoy's average measurements, as shown in Figures 5 and 6. From these graphs we can tell that the average step time was shorter than that of the wooden Tinkertoy created by the Cornell group. We found that our average step time was 0.34 ± 0.04 s and the Cornell group's step time of 0.47 s falls way outside of that uncertainty range [2]. We also found that our average step distance was 1.8 ± 0.3 cm, while the Cornell group measured a step distance of 1.3 cm [2]. Though our uncertainty was large in our distance measurements, the Cornell step length was still well outside of our uncertainty range.

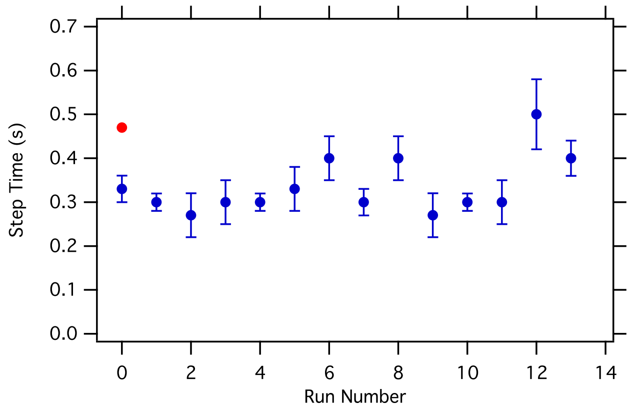

FIG. 5. Time length of steps versus the run number. The red dot on this graph represents the amount of time it took the Cornell group's walker to take a step, while the blue dots represent our data runs, which consist of separate times that the walker moved down the slope (varying from 1-3 steps per run). The error bars come from our estimated error in measurements. Our average step time was 0.34 ± 0.04 s, while the Cornell group's step time was 0.47 s [2]. This graph allows us to visualize the difference between these two walkers' behaviors.

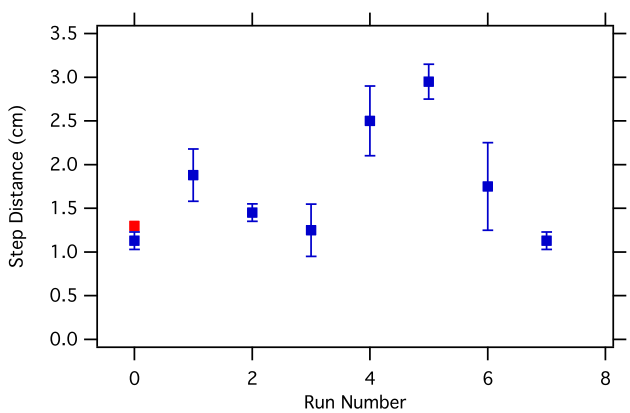

FIG. 6. Distance of each step down the slope versus the run number. The red dot on this graph represents the distance of the Cornell group's walker step, while the blue dots represent our data runs, which consist of separate times that the walker moved down the slope (varying from 1-3 steps per run). The error bars come from our estimated error in measurements. This plot has fewer points than Figure 5 because some of the walker shots were too angled to get a proper measurement. Our average step distance was 1.8 ± 0.3 cm, while the Cornell group's step time was 1.3 cm [2].

V. CONCLUSION

We created a passive walking robot out of plastic Tinkertoys, thus creating a physical representation of our computational simple walker model. Then, we filmed the walker moving down the slope multiple times and measured the length in time and distance of each step. From this we were able to compare the behavior of our walker to the Cornell group's wooden walker. The percent difference we calculated between our group and the Cornell group's step times and distances was 32% for both values. We discovered that though the designs were similar, the small differences led to noticeable changes in system behavior.

There are many reasons that our model toy behaved differently from the Cornell group's toy. One reason is that mass distribution due to the differences in materials effected the mobility of certain parts of the toy. Also, we used less slightly less mass on the feet balances than the Cornell group (due to material variances), so the balance differences likely caused the robot to walk differently. Lastly, we also did not put brass strips on the feet of our walker and used a wooden slope instead of a metal one. The inability to stand in any configuration like the Cornell's walker may have been integral to the walker stably walking down the hill, but it is unclear.

Throughout our research, we have learned that from simulation to physical reality, a passive walking robot is extremely sensitive to initial conditions and parameters. Without a specific combination of these things, the walker lacks stability and will either fall over or get stuck in a position. We've also come to our own realization that for this kind of system approximations and simplifications do not accurately predict or describe the system. The more complex the model is, the closer it gets to showing accurate predictions.

VI. FUTURE WORK

There are many components of this project that we were unable to complete because of the time restrictions of Junior IS. To expand upon the motion analyzed in this report, we would like to further analyze the motion of the walker. First, we would expand to looking at the hip and wobble motion of the walker. To do this, we would need to contact the Cornell group to see if they have any motion tracking data or side film from their passive walker that we could compare my videos to. Since that material was not readily available during this research period we were only able to look at step time and size.

We would also like to expand this project more into its education research side. We would acquire multiple sets of the Tinkertoy parts that we use for our walker, we would mark the necessary angles for placements, and time how long it takes for people of multiple age ranges to complete building it. From this we should be able to tell whether this is an efficient base to create an outreach or science day workshop out of it. If we can not modify it to allow for building to be used in outreach, we could create a fun interactive science day stop where kids can participate in the "Walker Challenge." This challenge would use our current walker, which can only take a few steps, and kids would compete to see who could get the walker to take the most amount of steps down the slope. This would be a great way to capture kid's attention during science day and could be placed somewhere that would redirect traffic as needed.

This work will be extended into a Senior IS project as well. We started this research project as a simple two-dimensional simulation, expanded it into a three-dimensional Tinkertoy model, and will take it to a full-scale dancer moving under the force of various gravities. Our hope is that this two year project will overall supply the scientific community with a greater understanding of the efficiency of different movements and the connection between physics and dance.

VII. ACKNOWLEDGEMENTS

I would like to thank Dr. John F. Lindner, Dr. Cody Leary, and the Wooster Physics Department for helping me with my experiment and providing the supplies necessary for this investigation. Additional thanks to my fellow Junior I.S. classmates and my favorite K-Pop bands for keeping my spirit high through the rough patches we all experienced the past few weeks.

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