Modeling Solar Sail propulsion in Spacecraft using Xcode Software

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Abstract

Stars emit solar radiation in the form of photons, which carry momentum and can apply a force to objects. Solar sails are designed to be large but lightweight, to take advantage of this force, as a means for propulsion; the sails deflect incoming photons for this purpose. This experiment models with Xcode, a coding environment, the dynamics of solar sails, in the presence of a star and a planetary body. When the angle θ between the normal of the sail and incoming photons is positive and less than $\pi/2$ then the sail is expected to have a positive net force and increase its distance away from the system. Likewise, when θ is negative and greater than -$\pi/2$ then the sail is expected to have a negative net force and decrease its distance between the bodies in the system. The model accurately demonstrates this. However, due to a divisional error with the gravitational forces when the sail's position approaches either celestial body's position then the program fails to realistically model the situation.

I. INTRODUCTION

Stars emit solar radiation in the form of photons, which are massless particles that do carry momentum. The change of the momentum of the photons has a force on objects, but is normally negligible. However, using very low mass, high surface area wings or sails, objects can capture solar radiation for a method of propulsion. The sails can reflect either the electric field, magnetic field, or both. In the case of both, the photons are reflected in a way analogous to a mirror, to harvest the change in momentum. An analogy can be made between traditional wind based sails used in seacraft and solar radiation powered solar sails used in spacecraft. In addition, techniques for the movement of different seacrafts in wind are also applicable to the movement of spacecraft utilizing solar sails. By changing the angle between the normal of the sail and the incoming photons to be negative, it is possible to sail in the direction of oncoming particles, which is initially counter-intuitive. This allows for considerable manipulation of the orientation of the spacecraft.

Solar sails allow spacecraft to significantly reduce their mass due to the reduction of fuel (except for fuel required to possibly power on-board electronics). Spacecraft utilizing this propulsion are permitted to take drastically different shapes, when compared to traditional rocket-based spacecraft. In an effort to increase the surface area of wings (to allow for more propulsion), there are several different types of shapes for wings, of which some may be modeled or new shapes conceived of. Solar sail based crafts are also able to reach significantly higher velocities than rocket-based crafts, due to their reduction in fuel-based mass. This would allow for solar sail based crafts to travel around our solar system farther and in possibly less time. Examples of solar sails based spacecraft have already been used. The IKAROS was one such solar sail based spacecraft.[1] This model aims at displaying the dynamics of solar sails in both a solar case, in which only the Sun and sail are present, and a planetary case, in which the Sun, Earth, and sail are present.

II. THEORY

Stars emit solar radiation in the form of photons, which do not have mass but carry momentum. This can be seen by Einstein's mass-energy equation:

$$E^2 = (m_0c^2)^2 + (pc)^2, \tag{1}$$

where E is the total energy, m_0 is the rest mass, c is the speed of light, and p is the momentum. In the case of photons, $m_0 = 0$. So even though photons do not have mass, they can still carry momentum. This change in momentum can be transferred to objects to produce a positive net force, due to the conservation of momentum. Solar radiation pressure is defined by this force felt by an object being struck with one or more photons. If the sail has a surface with area, A, then the time derivative of Eq.(1) is:

$$\begin{aligned} 2\frac{dE}{dt} &= 2\frac{d}{dt}(pc) = 2cf, \\ WA &= cf, \\ \frac{f}{A} &= \frac{W}{c}, \end{aligned} \tag{2}$$

where f is the force from the change in momentum, W is defined as the radiative flux. The result shows that the force per unit area is equal to the radiative flux divided by the speed of light. For this model the radiative flux, W, is assumed to be a constant value everywhere in the system. However, in actuality this is a function which depends on the distance between the sail and the star, which decreases as the sail moves farther away from the star.[2]

When a photon, from the solar radiation, strikes the sail it is assumed that it always reflects off of the sail, and is not absorbed into the sail or transmitted through the sail. Fig.(1) clearly shows the force diagram of the photon interaction with the sail. The incident photons, shown by vector *SR-I*, strike the sail with an angle, θ, from the normal of the sail. Then the photons reflect with a symmetric angle to the normal of the sail, shown by vector *SR-R*. Since the angle of a photon is symmetric with respect to its incident and reflected directions, the tangent components cancel. This then, produces a net force in the direction negative to the normal of the sail. If an area element, dA of the sail is considered, then the force of the element is described by:

$$F_p = -\frac{2WdA}{c}\cos^2(\theta)\hat{n}, \tag{3}$$

where F_p is the total forward force felt by the sail due to the solar radiation pressure and $\hat{n}$ is the unit normal of the sail. For the purposes of this model, W and dA were set equal to one.

The unit normal had to be found for this model to accurately display the sail's dynamics, with *just* a dependence on angles and not on the unit normal. So, using Fig.(1) again, the Sun is defined to be the origin of the system and the distance between the Sun and the sail is defined to be r_{SS}. The angle between r_{SS} and the x-axis is defined to be β. The length of the sail is defined as the vector $\vec{l}$, with the angle it makes with respect to the x-axis being ϕ. By removing the $\hat{n}$ dependence the force equation becomes:

$$F_p = -\frac{2WdA}{c}\cos^2(\theta)\begin{pmatrix} -\sin(\phi) \\ \cos(\phi) \end{pmatrix}. \tag{4}$$

The sail also experiences forces due to the gravity of the Sun and the planetary body, Earth. A complete force diagram can be seen in Fig.(2), with the Sun defined as the origin (the sizes of objects in the diagram are *not* drawn to scale). The equation for the gravitational force exerted by the Sun, on the sail, is given by:

$$F_S = \frac{GM_S m}{d_{SS}^2}\hat{d}_S = \frac{GM_S m}{d_{SS}^3}\vec{d}_S \tag{5}$$

where F_S is the force exerted by the Sun on the sail, G is the gravitational constant, M_S is the mass of the Sun, m is the mass of the sail, and d_{SS} is the distance between the Sun and the sail.

Similarly, the equation for the force exerted by the Earth on the sail is given by:

$$F_E = \frac{GM_E m}{d_{ES}^2}\hat{d}_E = \frac{GM_E m}{d_{ES}^3}\vec{d}_E \tag{6}$$

where F_E is the force exerted by the Earth on the sail, M_E is the mass of the Earth, and d_{ES} is the distance between the Earth and the sail.

Therefore, the total force equation for the dynamics of the sail at any point in the system is defined by the sum of all forces:

$$\vec{F}_{net} = \vec{F}_S + \vec{F}_E + \vec{F}_p. \tag{7}$$

For this model the Sun is in a fixed position at the origin and does not move due to forces from the the Earth or sail, as their masses are assumed to be too small for the forces to be significant. In addition, the Earth is assumed to have a perfect circular orbit around the Sun and is also not affected by the sail, since its mass is too small. The equation mapping the Earth's movement is given by:

$$\vec{r}_E = d_{ES}(\cos(\omega t)\hat{x} + \sin(\omega t)\hat{y}), \tag{8}$$

where $\hat{r}_E$ is the position vector of the Earth, d_{ES} is the distance between the Earth and the Sun (kept at a constant), t is the time, ω is defined as:

$$\omega = \frac{2\pi}{T}, \quad T = \sqrt{\left(\frac{4\pi^2}{M_S M_E G}\right) d_{ES}^3} \tag{9}$$

where T is the period of the Earth's orbit.

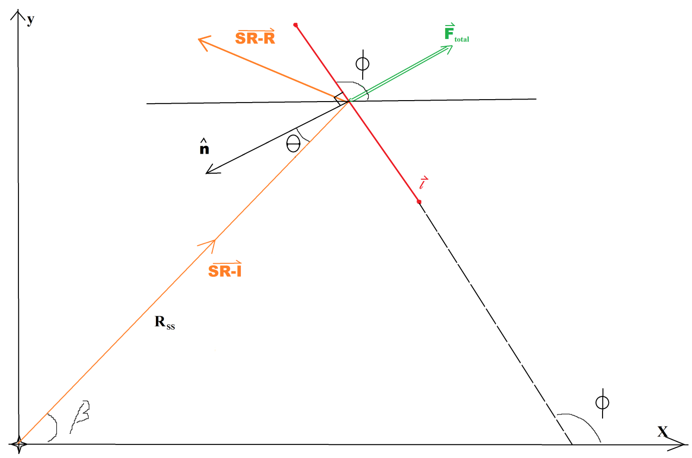

FIG. 1. A zoomed in force diagram for the solar sail in red, with length l; *SR-I* is the incoming, incident solar radiation, which reflects off of the sail, *SR-R*; n is the normal of the sail and θ is the angle between the normal of the sail and the incident solar radiation; β is the angle of the incident solar radiation with respect to the origin (i.e. the Sun); F_{total} is the total force produced from the change in momentum of the photon and is direction negative to the normal of the sail. r_{SS} is the distance from the origin to the sail.

III. IMPLEMENTATION

This model uses Xcode, a coding environment, and the Obj. C language in order to function. The Appendix section displays the main code for the simulation and should be used in conjunction with further explanation.

Initially the code implements and initiates constants as specific values before the program is actually displayed to the user. This is seen in the awakeFromNib method. The software the program uses could not interpret very large numbers (in the larger than 10^6 range)

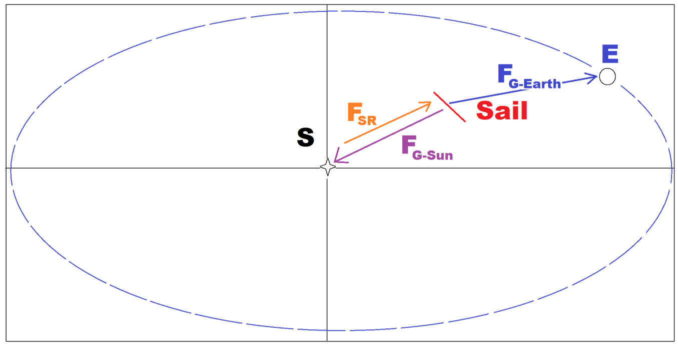

FIG. 2. A global force diagram of the system, where the Sun, S, is defined as the origin of the system, E is the Earth with the dashed blue line being a representation of its orbit around the Sun, the *Sail* is the solar sail with three forces acting upon it: blue for the gravity of the Earth, purple for the gravity of the Sun, and orange for the force due to solar radiation.

and very small numbers (in the smaller than 10^{-4} range). As a result, constant values do not necessarily have an accurate number attached to them. For instance, the mass of the Sun is initialized as 10 and the Earth 1, but in actuality the mass of the Sun would be approximately 333,000 times the mass of the Earth. In addition, several constants have the ability to be changed by the user via a slider or a text box. The origin is also changed to the Sun's position of the system.

The implementation method calls for the program to reset itself. Here in the reset method, the positions and velocities of the Earth and sail are implemented based on the starting conditions of the program. The program has two sets of starting conditions, called the mode: when there is no Earth present, and when Earth is present. These conditions can also be changed while the program is running, by doing so can the user can implement either the solar case method or the planetary case method directly. In addition, the trail which can be seen in Fig.(3), is reset and initiated in the reset method. The reset method can also be implemented by the user manually, in order to restart the program with changed variable parameters. Further, there is another reset method (fullreset), which completely restarts the program with the original initialized parameters.

In the drawRect section of the code, the program draws all aspects of the program. This sets the background, then draws the Sun, with a scalable radius, then the Earth (if applicable) again with a scalable radius, then the sail with a scalable length, and then the positional trail of the sail and the normal of the sail.

In the animate section of the code, the program animates the drawing, as long as the program is not paused. The animate method calls for an animateStep method, which runs a method based upon the initial starting conditions (or when those conditions are changed by the user). The particular animation method used depends on the mode, which again can be changed whilst the program is in use. The first animation method only considers the case when there is no Earth. This starts by computing the acceleration due to the gravity of the Sun acting on the sail, following Eq.(5). Then the computes the acceleration due to the solar radiation pressure and the unit normal vector, following Eq.(4). Finally this method combines these two acceleration and then integrates using Euler-Cromer integration to obtain the new velocity using the old position then the new position using the recently computed velocity. The second animation method follows the same procedure as the first animation method, except this also computes the acceleration due to the gravity of Earth and Earth's circular orbit animation.

IV. RESULTS AND ANALYSIS

Under Eq.(4) the sail's interaction with light should yield a positive net velocity away from the Sun provided that θ is positive and less than $\pi/2$. This behavior can be seen in Fig.(3). As the sail progresses in time, its position is mapped by a trail, which produces spirals with only the Sun in the program. When the sail's angle is negative this produces inwards spiral towards the Sun, which can be seen in Fig.(4). However, when the sail's position equals the Sun's position (this is also true for when it equals the Earth's position), then under Eq.(5 or 6) the distance becomes zero, which forces the program into a division by zero error. This error causes the sail to have a large force from gravity away from the system. When the Earth is placed into the system, then the sail is influenced by the Earth's gravity and therefore follows Eq.(7), this can be seen in Fig.(5). These orbits, particularly from Fig.(5), conform with similar orbits for previously documented Earth-Sun systems.[3]

V. CONCLUSION

Solar sails are designed to capture as much solar radiation pressure as possible as a means for propulsion; the sails deflect incoming photons for this purpose. This experiment models with Xcode, a coding environment, the dynamics of solar sails in the presence of a star and a planetary body. When the angle, θ between the normal of the sail and incoming photons is positive and less than $\pi/2$ then the sail is expected to have a positive net force and increase its distance away from the system. Likewise, when θ is negative and greater than $-\pi/2$ then the sail is expected to have a negative net force and decrease its distance between the bodies in the system. The model accurately demonstrates this, however, due to a divisional error with the gravitational forces when the sail's position approaches either celestial body's position then the program fails to realistically model the situation.

VI. REFERENCES

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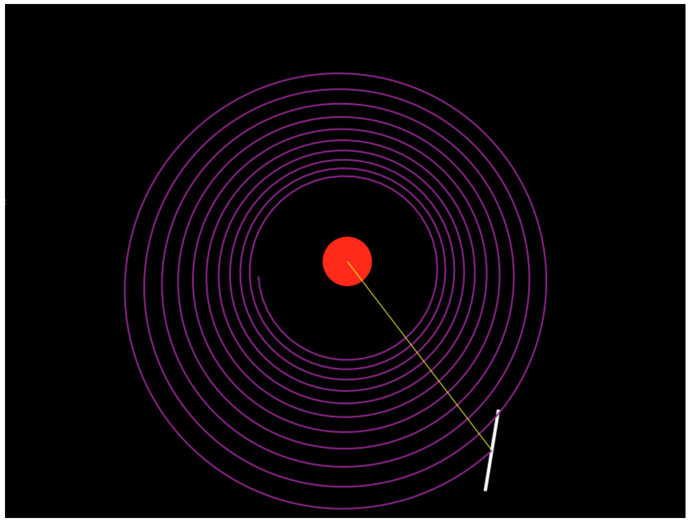

FIG. 3. This picture shows when θ is positive and therefore shows the sail spiraling away from the Sun. The sail is white; Sun, red; with a spiraling line, purple, detailing its prior position. The yellow line is the normal of the sail, in the direction of the Sun.

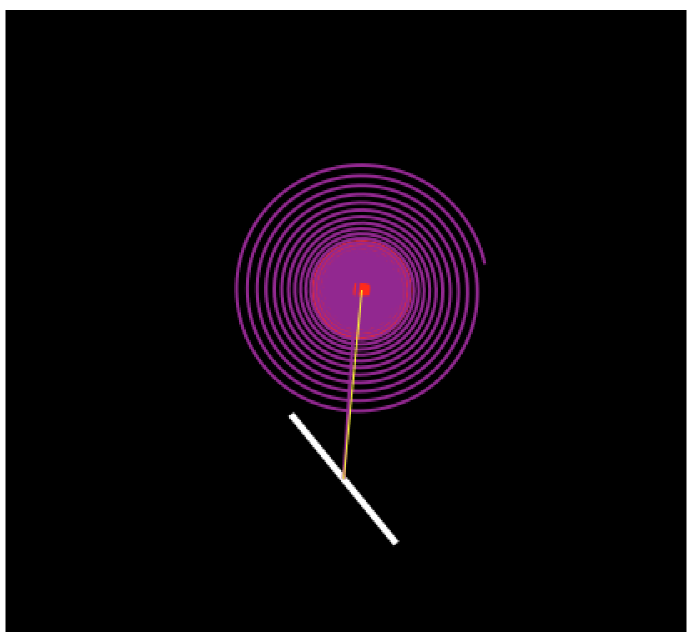

FIG. 4. This picture shows when θ is negative and spiral towards the Sun. This picture also shows the divisional error with the gravitational forces which causes the sail the gain an infinite force away from the Sun, i.e. the origin, there is a computational error and the sail leaves the system. See Fig.(3) for definitions.

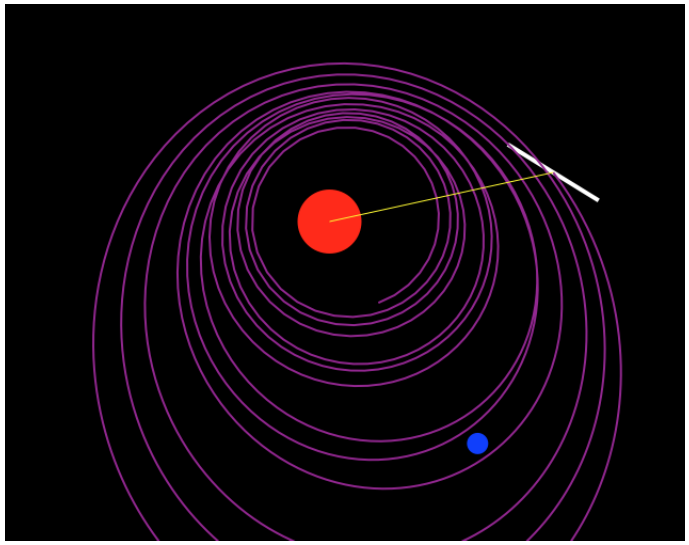

FIG. 5. This picture shows another possible orbit of the sail in the planetary case, with the Earth in blue. See Fig.(3) for definitions.