Measuring Energy Dissipation Within Various Granular Matter by Means of Hacky Sacking

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Abstract

This experiment used five identical hacky sacks to measure the energy dissipation between various granular matter. Each sack was cut open, emptied, and filled with the same volume of either small plastic beads, couscous, beans, salt, or rice. The sack was dropped from the same height each trial and the maximum height traveled by the sack after it was kicked was recorded via the particle tracking software Tracker. The maximum heights were then compared to that of a perfectly elastic collision under the same conditions which was replicated by use of a ping pong ball. Using the comparison of the average maximum heights reached for each granular matter to that of the elastic collision, the energy dissipation could then be calculated. The salt was found to have the least energy dissipation as 38 % of its energy was lost. That was small in comparison to the granular matter which was found to be the most dissipative, the small plastic beads, which had 62 % of its energy dissipated. The couscous, beans, and rice had 52 %, 46 %, and 42 % of their energy dissipated respectively. The results have no basis of comparison as this has never been tested before, so the accuracy is unknown, however, the data could be improved by using more automation or additional sensors which could produce more precise data.

1 Introduction

The concept of kicking a ball up into the air repeatedly has been around for centuries. However, it was not until John Stalberger paired up with Mike Marshall in 1972 to further explore the hobby Stalberger had learned from a friend that the sport of hacky sacking was officially developed. The two experimented with different shapes, cover materials, and fillings for their kicking object. Buttons, rice, seeds, and rocks were all used in the initial attempts at making the perfect footbag [1]. The coverings for hacky sacks still vary today; however, the bounce and quality of the sack depend on the granular matter that fills the inside.

The reason the granular matter is important to the sport of footbag is because of the energy dissipation. Interactions between grains are inherently inelastic, some energy is lost in each collision [2]. Granular matter is a subcategory of soft matter which are units that are significantly larger than an atom but much smaller than the overall dimensions of the sample. During the sacking process, the granular matter within the sack collide and scrape past one another. These collisions generate significant dissipation during flow or deformation. Kinetic energy, for example, due to sound vibrations, is rapidly absorbed into the internal degrees of freedom of the constituents [3]. Different size and shaped granular matter will have different interactions between individual particles which will lead to different energy dissipations in a single kick in hacky sacking.

Granular matter can have properties of both solids and liquids. An individual grain, such as a single marble, dropped onto a glass plate, will bounce for quite a while. Identical marbles, loosely filled into a sack, will stop dead on the plate. This strikingly different collective behavior arises from the exceedingly large number of rapid inelastic collisions among neighboring grains [2]. When the granular matter is contained in a sack, their collision behavior resembles that of a solid. However, when the individual particles are unconfined, they bounce similar to how a liquid would react. During a collision, a liquid will splash, deposit, and bounce which is how the singular grain of granular matter behaves [4]. In this way, granular matter has the ability to stop as if it were a solid, but also bounce and flow similar to the behaviors of a liquid. Having properties of both solids and liquids, granular matter is a heavily researched subject.

The difference in energy dissipation as a result of different granular matter can be measured by means of a hacky sack. Filling the hacky sacks with the different grains will allow us to measure the respective level of energy dissipation.

2 Theory

We have learned by studying the physical properties of granular matter that they have the ability to work as dampers. When a container was partially filled with a mass of grains, during a collision, the kinetic energy of the container is partially transferred to the grains, the rebound is damped and the fast energy dissipation through inter-particle collisions and friction decreases the bouncing time dramatically [5]. In a collision with a ball filled with granular matter, mechanical energy is not conserved due to collisions of the grains.

Granular matter is a broad category in which the grains can come in a variety of shapes and sizes. The rate of damping and the energy dissipation of the specific type of granular matter changes depending on those shapes and sizes. Particles that are non-spherical complicate the mechanics of the collision opposed to spherical particles and the outcome of an impact event depends on the particle shape and orientation [6]. Tests have shown a sheared granular matter, or granular matter that is not perfectly uniform and cut, has a higher collision rate of the grains during impact than that of a spherical granular matter [7]. The collisions can be idealized when imagining perfectly inelastic collisions of hard spheres which can easily be modeled gaseous states of granular matter. In this model, the grains only touch during the collision, allowing for the energy dissipation of the granular matter to be found by calculating changes in kinetic energy in the system [8].

To simplify the energy dissipation of the granular matter within the hacky sacks, we can use the theory behind analyzing that of the idealized gaseous states of granular matter and calculate changes in kinetic energy. We know from introductory physics that the total energy in a system before and after an event can be written by

$$\Delta K + \Delta U + \Delta E_{int} = 0, \tag{1}$$

where K is kinetic energy, U is potential energy and E_{int} is the change in energy due to a non-conservative force [9]. Expanding the equation to initial and final conditions gives

$$K_f - K_i + U_f - U_i + \Delta E_{int} = 0. \tag{2}$$

The potential energy immediately before and after the collision will go to zero. Therefore, we only consider the kinetic energies incorporated in our system. We have that of the sack K_s and that of the foot kicking the sack K_f. In general, $K = 1/2\ mv^2$ where m is the mass of the object and v is velocity, we have

$$\frac{1}{2}[m_s v_{sf}^2 + M_{eff} v_{ff}^2] + \Delta E_{int} = \frac{1}{2}[m_s v_{si}^2 + M_{eff} v_{fi}^2], \tag{3}$$

where M_{eff} is the mass effective from the foot on the system during the kick. The mass effective in the system must be a calculated value as finding the mass of only a foot is impossible by experimental means. We use the results of a study which measured the force of impact on the bottom of the foot while walking, crouch walking, and running [10]. The effective mass

(a) (b) (c) (d) (e)

Figure 1: The measured amount of each raw granular matter used for the experiment. Materials included are a) small plastic beads, denoted as "normal", b) raw couscous, c) dry beans, d) course kosher salt, and e) dry brown rice.

in these activities calculated the M_{eff} by using the impulse-momentum method which says impulse is equal to the change in momentum. The magnitude of M_{eff} is a percentage of the body mass M_b and was found to be $5.6\% M_b$ [10]. Applying this to our system, $M_{eff} = 3.416$ kg.

The average mass of a hacky sack is less than 2% of the effective mass of the foot in this scenario, therefore, the energy put into the system by the foot is much larger than the slight dissipation of energy in the sack. The energy dissipation within the granular matter is still present and is a measurable variable, but more precise instruments such as higher speed cameras or force sensors would be needed to detect this small number within the large system. Using particle tracking as the method of analysis, the energy dissipation is so small the results would not have been able to have been calculated or would not have been accurate due to the mechanical energy of the foot being so much larger than that of the sack. Therefore, we turn to a different method to find the energy dissipation.

Using a different approach to our analysis, we can look at the maximum height that the sack travels. We use the same principles applied in Eq. 2, to investigate a perfectly elastic collision. When looking at the point after the collision of the maximum height, h_{max}, that the sack travels, there is no kinetic energy in the system, thus K_f goes to zero. The moment just before the collision is the initial energy, thus initial potential energy U_i also goes to zero. By expanding K_i and U_f we can see

$$\frac{1}{2}[m_s v_{sf}^2 + M_{eff} v_{ff}^2] = m_s g h_{max}$$

$$h_{max} = \frac{1}{2 m_s g}[m_s v_{sf}^2 + M_{eff} v_{ff}^2] \qquad (4)$$

Between each trial, the mass of the sack changed slightly, but the change is never greater than 0.04 kg, so m_s can be treated as a constant. Also, we assume the sack was kicked with the same amount of force each time so the kinetic energy originating from the foot can also be treated as a constant. Each sack was dropped from one meter above the ground, so by ignoring air resistance, we can assume the velocity of the sack just before impact was also approximately the same, as the height of impact was approximately the same each kick. Therefore, in a perfectly elastic collision, everything h_{max} depends upon is a constant. By comparison, we can take the h_{max} for each hacky sack and divide it by that of an elastic collision to find the percentage of energy remaining in the sack. That implies, to find the percentage of energy that dissipated for each material, we subtract that ratio from one and multiply by 100:

$$100\left(1 - \frac{h_{max_{sack}}}{h_{max_{elastic}}}\right) = \% \text{ of } E_d. \qquad (5)$$

This method provides a practical way of finding the energy dissipated for each granular material tested.

3 Procedure

In order to measure the energy dissipation amongst various granular material within a

Table I: The granular matter with the corresponding masses used for each sample of grains.

Label	Granular Matter	Volume ±1 (ml)	Mass of Sack ±.05 (g)
N	Plastic Beads	65	39.33
C	Couscous	65	51.58
B	Beans	65	45.04
S	Salt	65	71.43
R	Rice	65	52.28

Figure 2: Each of the five hacky sacks filled with different granular matter, labeled and sewn shut.

hacky sack, different sacks must be acquired to be used for testing. Five identical hacky sacks were obtained and cut open to be filled with their respective granular matter. The granular matter chosen to be tested was first the small plastic beads that came in the sack, then other materials that filled the remaining four sacks were couscous, dry beans, salt, and dry rice. An image of the raw materials used as fillings can be seen in Fig. 1.

The same volume of matter was placed into each sack and the corresponding mass was then recorded. The matter with its corresponding mass within the sack can be seen in Table I.

Each sack was then sewn shut and labeled with a marker to then be used for data collection. The five sewn sacks can been seen in Fig. 2

To conduct data collection, a Casio Exilim camera was set up on a tripod and centered on a blank wall where it remained for the duration of data collection. A meter stick is placed in the frame of view in the same plane as where the sack is being dropped from so in the video we have a reference of one meter. Each video collected consisted of lining up in front of the camera, dropping the sack from one meter above the ground, and kicking it once with the inside of my right foot. The sack then rebounded up into the air and fell following the motion of a body in free fall. The distance the sack traveled from the dropping spot along the x and y axis is recorded after each drop. The orientation of my body was the same every time to ensure consistency with each kick. Feet were aligned to tape on the ground and the sack was dropped over a drop spot which was designated to be the origin of the XY plane. An aerial view of the markings on the ground that were used for alignment can be seen in Fig. 3.

A trial of five kicks was first recorded of kicking a ping pong ball. This is to simulate a perfectly elastic collision to provide a comparison for our data. Each hacky sack is kicked five different times with the landing spot recorded manually measuring the change in x and y from the drop spot. The videos from the drops were then transferred to be analyzed in the particle tracking software Tracker. In Tracker, we can use the particle tracking features to calculate

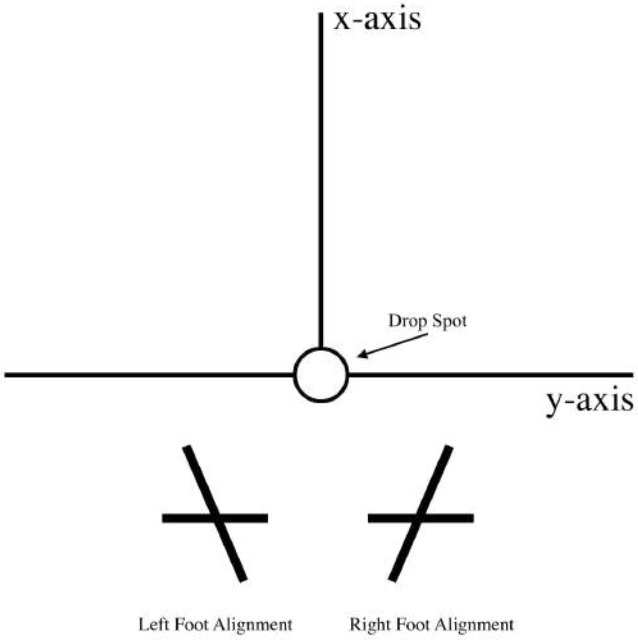

Figure 3: A view from the kicker's perspective of the ground to align themselves for each kick.

the velocity of the sack and the foot just before and just after the collision and find the maximum height the sack reaches in a single kick. A series of the images from particle tracking just before collision, during collision, and just after can be seen in Fig. 4.

Each video for the trials must be tracked in order for velocities and heights to be recorded. With the numbers recorded, we can find the energy dissipation of the granular matter.

4 Data & Results

Five trials using each granular material were tested and the maximum height and velocities of the foot and the sack were tracked and recorded. As a basis for comparison, we measured the average maximum height reached by a ping pong ball which will be considered that of a perfectly elastic collision in this scenario.

The average velocity of the foot coming into the collision was 1.533 m/s with a standard deviation of 0.406 m/s. This number was considered a constant throughout each trial in order to complete the analysis.

Using the initial methods of analysis we intended on using, we could calculate the kinetic energy of the sack just after impact using the velocities measured by tracking, and compare it to that of the ping pong ball to determine energy dissipation. The average kinetic energies for each granular matter were calculated and plotted and compared to that of the ping pong ball. This can be seen in Fig. 5, a plot made in analysis software Igor.

The results from this plot are surprising if one does not consider the mass of the different objects. The mass of the ping pong ball is approximately 20 times smaller than the average mass of the sacks, thus its average kinetic energy after each collision is much smaller than those of the sacks. We must use maximum heights for our analysis as the magnitude of the mass of the ping pong ball is not significant.

(a) (b) (c)

Figure 4: A series of screenshots showing the process in Tracker to track the sack and my foot throughout the entire collision. Shown is a) the moment before the collision, b) the moment the objects are in contact, and c) the moment just after the collision.

By looking at the maximum height of the object being kicked, we are neglecting the x and y motion of the object. With this stipulation, we ruled out all data points in which the majority of the motion of the sack or ball was not upwards. This only occurred in two of the trials using ping pong balls, as the trajectory of the ball was much harder to keep straight.

The maximum height was found for each trial of each material, and the results were averaged to then find the average energy dissipation for each granular matter. The results can be seen in Table II.

Trends in the data can be seen by making a bar graph of the average maximum height for each granular matter and spacing them out for an easier comparison. As seen in Fig. 6, the general energy dissipation can be visualized by the stacks of data. The ping pong balls maximum heights were much larger than the other materials as this was the elastic collision. The plastic beads that came in the sack originally had the most energy dissipation, which can be seen in the plot by their maximum height being the smallest of all of the granular matter, much lower than those of the ping pong ball's. Rice and salt showed lower levels of energy dissipation in the collision. Beans and couscous had roughly the same amount of energy dissipation and their maximum heights are scattered in the middle.

The small plastic beads that came in the original hacky sack proved to be the most dissipative granular matter losing 62 % of its energy, whereas salt proved to be the most efficient granular matter and had the least amount of energy dissipation, 38 %, during a kick.

5 Conclusion

The purpose of this experiment was to measure the energy dissipation amongst various granular matter by means of kicking a hacky sack. This goal was achieved by filling identical hacky sacks with the different granular matter tested: small plastic beads, couscous, beans, salt, and rice. The sack was kicked with the same velocity after being dropped from a standardized height. The maximum height the sack reached after the kick was then compared to the maximum height reached by a ping pong ball under the same conditions, considered to be a perfectly elastic collision. The percentage of energy dissipated could then be calculated and averaged for each granular matter. The salt conserved the most energy, only losing 38 % of its energy and the small plastic beads were the least conservative, losing 62 % of its energy in the collision.

This experiment gives us insight as to which granular materials conserve the most energy

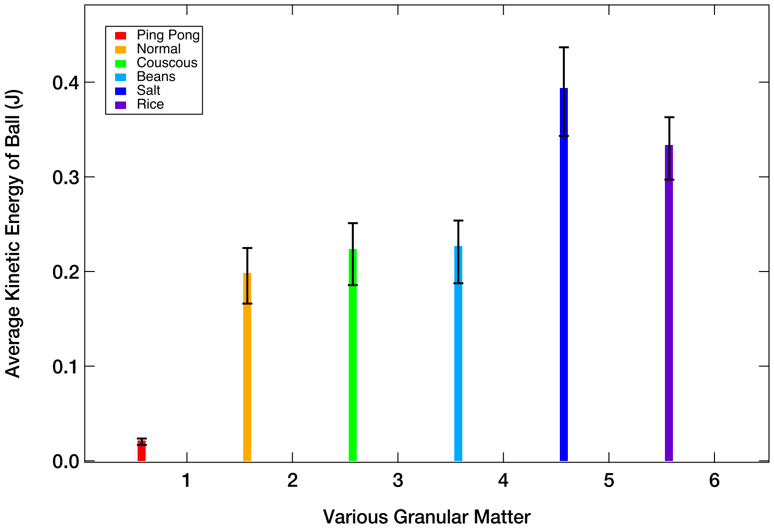

Figure 5: The average kinetic energy of each sack and the ping pong ball after impact.

Table II: The calculated percentage of energy dissipation within each granular matter based on the average maximum height reached by the sack in comparison of that of the ping pong ball.

Granular Matter	$h_{max} \pm 0.05$ (m)	$\frac{h_{max_{sack}}}{h_{max_{elastic}}}$	$\%E_d$
Ping Pong	1.26	1	0 %
Normal	0.49	0.39	61 %
Couscous	0.61	0.48	52 %
Beans	0.69	0.55	45 %
Salt	0.78	0.62	38 %
Rice	0.73	0.58	42 %

and which do not. The energy is lost in a collision is due to the inter-particle interactions within the sack, so we can infer the differences in energy dissipation is partially due to the different shapes and sizes of the grains themselves. Smaller sized and rigid grains could lead to more interactions between the particles which would lead to a greater energy dissipation compared to a spherical, smooth grain. As the same volume of granular matter was placed into each sack and the grains of each matter were different sizes, this implies a different number of grains in each sack. The different number of grains also leads to a different number of collisions between particles which causes a slight discrepancy in energy dissipations between granular matters.

This experiment could be pursued further to draw more conclusions about granular matter. We could do testing again, but this time fill each sack with the same number of grains rather than the same volume of granular matter. This could assist in controlling for the number of inter-particle interactions which could give us a better idea of the differences in energy dissipation within each granular matter. This experiment gives us a start to understanding what is happening in a collision at the particle level of granular matter, but more experimenting would give us more precise information.

There are also large margins of error in this analysis. There were many assumptions made to optimize the system to simplify the calculation of energy dissipated. To possibly improve

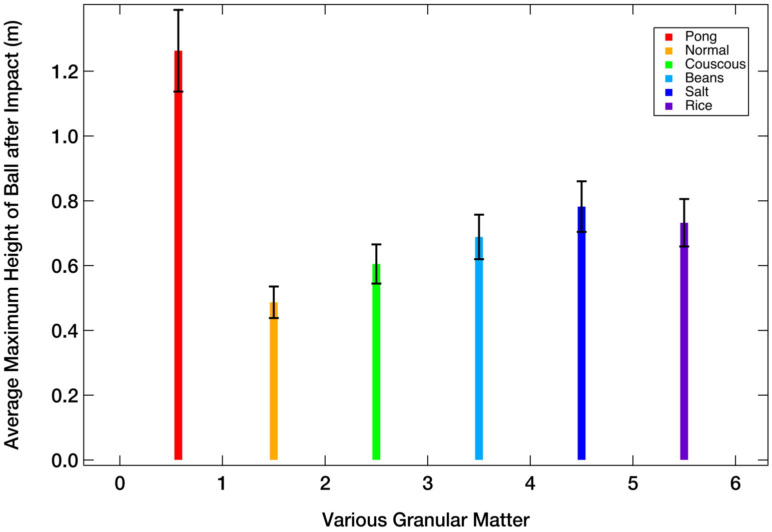

Figure 6: The average maximum height each granular matter reached plotted to be easily compared with one another.

these results, the kick generated by the human body could be replaced by a contraption that generates the same force every repetition. Also, a force sensor could be placed on this contraption, so the exact force going into the hacky sack each kick can be measured, thus allowing for the results from Eq. 3 to be used to calculate the energy dissipated. This contraption would also kick the sack in a consistent spot and that spot could be manipulated to allow for each kick to travel only straight up. This would reduce the error in finding the maximum height reached by each sack as they would no longer be traveling a significant distance in the XY plane. That would justify the optimizations assumed in this analysis to use Eq. 5 which calculated the energy dissipation of each material. Using a machine to do the kicking instead of a human body would reduce error made in this experiment and further justify the assumptions which made the variables such as velocity of the foot and distance traveled in the x and y directions reduce to constants.

We achieved our goal by measuring the energy dissipated in various granular matter by comparing the maximum height reached after a kick to that of an elastic collision. The accuracy of results of this lab are currently unknown as no one is known to have previously measured this before, however, we do know improvements could be made to the results. By changing some of the techniques and equipment used as described above, the error in our calculations could have been reduced and given us an even better idea of the exact level of energy dissipation in the granular matter tested.

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