

Experimental and Simulated Explorations of Feynman's Disk Paradox

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In this experiment I attempted to create a physical model and a virtual model to visually show the Feynman disk paradox in action. Richard Feynman's disk paradox was a thought experiment that said if a free-floating disk of charges surrounded a solenoid with current flowing through it, the disk would rotate when the current was shut off due to a transfer of angular momentum from the crossed electric and magnetic fields. I created two different physical models using a solenoid to create a magnetic field and suspended metal spheres that had positive charge on them. In my second physical model, my calculations showed that the charged sphere would rotate a maximum of 1/100,000 degree which was not visible to the naked eye or measurable using available equipment. This led me to create a virtual model using Mathematica, which showed that the angular momentum field circulated clockwise around the axis of the solenoid and was created by the crossed electric and magnetic fields when the solenoid's current was interrupted.

INTRODUCTION

Richard Phillips Feynman was a theoretical physicist from Queens, New York. Feynman is best known for his contributions in quantum electrodynamics, particle physics, superfluidity, and quantum mechanics. He even assisted in the creation of the atom bomb during World War II. In 1965, Feynman received a Nobel Prize in physics for his development of a space-time view of quantum electrodynamics, along with Julian Schwinger and Sin-Itiro Tomonaga [1].

Richard Feynman realized a connection between electromagnetic induction and conservation of angular momentum, as explained in his *Feynman Lectures*. With this connection, Feynman developed a paradox (known as the Feynman Disk Paradox) that seemingly defied the law of conservation of angular momentum. In the theoretical paradox Feynman described, a coil of wire with current flowing through it (a solenoid) was surrounded by an insulating disk that was filled with equally spaced electrostatic charges. In this model friction was neglected so that the disk could move freely. Feynman theorized that if the current through the solenoid was interrupted somehow, this would cause a change in the magnetic flux. Faraday's law of electromagnetic induction states that the electric field induced on the charges exerts a force on them. Feynman predicted that this force would act on the disk of charges and cause it to rotate. The paradoxical aspect of this is that when the current is flowing through the solenoid, the disk is at rest. According to the law of conservation of angular momentum for the disk, we would expect the disk to stay at rest when the current is interrupted. The solution to this paradox is that the static electromagnetic fields have stored angular momentum, as explained further in the *Theory* section of this paper. As the electromagnetic field loses angular momentum, the disk gains angular momentum and rotates [1, 2].

In my experiment, I created two physical models us-

ing solenoids, metal spheres, and DC power supplies and one virtual model using Mathematica in an attempt to visualize and test the Feynman disk paradox.

THEORY

In Feynman's disk paradox, if the current through the solenoid is interrupted, the change in magnetic flux will induce a circulating electric field. To unravel the Feynman disk paradox, I first needed to look at Faraday's Law that says the circulation of the electric field around a closed loop is negative the time rate of change of the magnetic flux through a surface bounded by the loop. Mathematically this is written as

$$\Gamma_\epsilon = -\dot{\Phi}_B = -\frac{d}{dt}\Phi_B, \quad (1)$$

where Γ_ϵ is the circulation of the electric field and $-\dot{\Phi}_B$ is the first time derivative of the magnetic flux. In general, a surface with area a has a total magnetic flux of

$$\Phi_B = \iint \vec{B} \cdot d\vec{a}, \quad (2)$$

through the surface. The circulation of the electric field for a closed loop is expressed as

$$\Gamma_\epsilon = \oint \vec{\epsilon} \cdot d\vec{l}, \quad (3)$$

which says that the circulation of the electric field is equal to the closed integral sum of the electric field $\vec{\epsilon}$ over small changes in the length of the loop $d\vec{l}$. Circulation is the amount of emf along a closed loop. Substituting Eq. 2 and Eq. 3 into Eq. 1, it is clear that

$$\oint \vec{\epsilon} \cdot d\vec{l} = -\frac{d}{dt} \iint \vec{B} \cdot d\vec{a}. \quad (4)$$

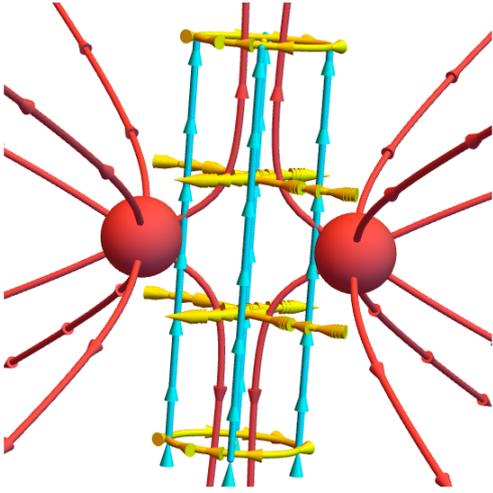


FIG. 1: Feynman disk paradox schematic. This Mathematica model shows the charges and their corresponding electric field lines in red and pointing away from the positive charges, the magnetic field lines in blue and pointing up, out of the solenoid, and the momentum field lines in yellow.

In Feynman's disk paradox, a solenoid is surrounded by a floating circular disk containing point charges. Solving the integrals in Eq. 4 for this system gives

$$\varepsilon 2\pi r = -\frac{d}{dt}(B\pi R^2), \quad (5)$$

where r is the distance between the center of the solenoid and the point charge and R is the radius of the solenoid. By taking the time derivative of $B\pi R^2$ and dividing both sides of Eq. 5 by π , I found that

$$\varepsilon 2r = -\dot{B}R^2. \quad (6)$$

Figure 1 shows the magnetic, electric, and momentum fields of the Feynman disk paradox corresponding to these equations.

Solving Eq. 6 for the magnitude of the electric field ε shows the relationship between the magnitude of the induced electric field and the time varying magnetic field. It also shows that ε is proportional to R^2 and inversely proportional to r . This relationship is written as

$$\varepsilon = \frac{-\dot{B}R^2}{2r}. \quad (7)$$

Feynman's disk paradox predicted that the circulating electric field ε , induced by a change in magnetic flux, would exert a force on the point charges located in a floating disk around a solenoid. As the force rotates the disk of charges, a torque is produced. The torque τ on the point charges due to a circulating electric field can be expressed as

$$\tau = |2Q\varepsilon|, \quad (8)$$

where Q is the amount of charge on an individual point charge in the disk. Substituting the results for ε in Eq: 8,

$$\tau = \frac{|\dot{B}|R^2}{r}Q. \quad (9)$$

Ampere's law relates the magnetic field around a closed loop to current flowing through the loop. Ampere's law can be generalized for solenoids so

$$B = \mu_0 n I_0, \quad (10)$$

where the magnetic field B of the solenoid is equal to the product of the turn density n , the initial current flowing through the solenoid I_0 , and the permeability of free space μ_0 . The turn density is defined as the number of turns in the solenoid divided by the length of the solenoid. The permeability of free space is the constant, $4\pi \times 10^{-7}$ T/A·m. To fulfill Eq. 9, the first time derivative of the magnetic field is needed. The first time derivative of Eq. 10 is

$$\dot{B} = \mu_0 n \dot{I}_0, \quad (11)$$

where \dot{I}_0 is the change in the current flowing through the solenoid over time. Substituting Eq. 11 into Eq. 9 provides an explicit formula for torque that shows that torque is proportional to the turn density of the solenoid, the change in current flowing through the solenoid over time, the square of the radius of the solenoid, and the charge on the point charges in the floating disk. This relationship is given as

$$\tau = \mu_0 n \dot{I}_0 \frac{R^2}{r} Q. \quad (12)$$

In my experiment, I used a sphere instead of point charges in a disk to test the Feynman disk paradox. For the charge Q on the sphere, I used the equation

$$Q = CV. \quad (13)$$

The variable C is the capacitance of the sphere and V is the voltage of the charged sphere. The capacitance C of the sphere is defined as

$$C = 4\pi\varepsilon_0 a, \quad (14)$$

where $\varepsilon_0 = 8.85 \times 10^{-12}$ F/m. Also in my experiment, the charged sphere was suspended by a torsion wire. The

variable, a , is the radius of the charged sphere. The torsion wire had a torsion constant $k = 1.37 \pm 0.02 \times 10^{-6}$ N/degree. The force F on the charged sphere is related to the torque by

$$F = \frac{\tau}{r}, \quad (15)$$

where r is again the distance between the spheres and the center of the solenoid. By substituting Eq. 15 into Eq. 9, the force can be rewritten as

$$F = \frac{|\dot{B}| R^2}{r^2} Q. \quad (16)$$

Using Hooke's law, $F = k\theta$, the angle θ at which the force from the circulating electric field causes the sphere to move can be found by

$$\theta = \frac{F}{k}. \quad (17)$$

When the current is interrupted, the charged sphere should rotate an angle θ and visualizing this angular displacement would verify Feynman's disk paradox [2–4].

PROCEDURE

Physical Disk Model

For the first phase of my experiment, I created a physical apparatus to test the Feynman disk paradox experimentally. To create the apparatus, I started with a hand-made solenoid with 715 turns of 22 gauge enamel-coated copper wire. I then had a ring created out of Plexiglas made to fit around the diameter of the solenoid. Fitted inside the Plexiglas ring were two 1 inch diameter solid steel balls. They were positioned 180° apart on the ring. A solid steel bar was then inserted into the center of the solenoid in order to increase the magnetic field produced by the solenoid. Three holes were drilled into the Plexiglas ring and string was used to suspend the ring, centered around the solenoid shown in Fig. 2.

After the apparatus was set up, the enamel coating on the tips of the solenoid wires was removed using sandpaper and the wires were connected to a DC power supply to provide a current through the solenoid and a corresponding magnetic field.

In order to test the Feynman disk paradox, the steel balls in the Plexiglas ring needed to have a charge on them. To do this, I transferred charge to the steel balls from a Van de Graaff generator using a wand I created that had a wooden handle and a spherical metal tip. The charge supplied by the Van de Graaff generator was not measured. Immediately after placing charge on the steel

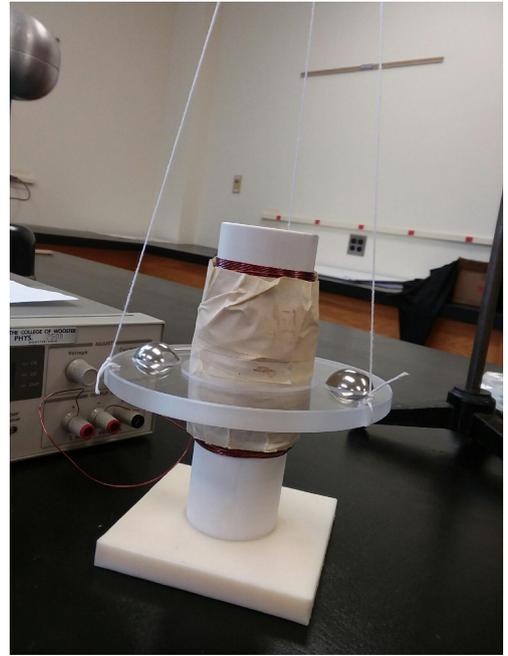


FIG. 2: Photograph of the disk model. A solenoid was surrounded by a Plexiglas disk. This disk was suspended using three strings that were tied around an overhanging metal stand. Inside the Plexiglas disk, the two steel balls were 180° apart to balance the disk.

spheres, the DC power supply was turned on and a current of 1.5 ± 0.1 A was sent through the solenoid. The entire set-up is pictured in Fig. 3. To fulfill the requirements of the Feynman disk paradox, the current was interrupted by shutting off the DC power supply to create a change in the magnetic flux and a circulating electric field. After testing the apparatus several times, no visible confirmation of the Feynman disk paradox was present. The disk did not spin when the current was interrupted. Initially, it was thought that this lack of visible rotation was because the strings caused too much resistance to the spinning of the disk and this model was eventually abandoned due to the uncertainty of the string tension and the uncertainty in the amount of charge transferred from the Van de Graaff generator to the steel spheres.

Physical Rod Model

A second apparatus was created with several improvements from the disk apparatus. This version used pieces from a Pasco scientific Model ES-9070 Coulomb Balance. The apparatus used a hollow, lightweight ball suspended by a sensitive torsion wire shown in Fig. 4. The apparatus was made so that if the torsion wire was twisted by the sphere moving, then a knob could be turned to bring the wire back to equilibrium. The knob provided the angle that the wire was turned in order to calculate

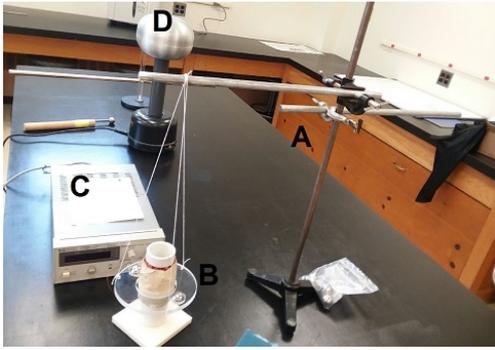


FIG. 3: Photograph of the equipment needed to test the disk model. (A) A ring was suspended by strings tied to a metal beam stand. (B) A Plexiglas ring contained steel balls. This ring was centered around a solenoid. (C) The solenoid was provided a current using a DC power supply. (D) A Van de Graaff generator transferred charge to the steel balls.

the force on the ball using Hooke's law $F = k\theta$. The same solenoid and DC power supply were used as in the disk model discussed previously. Due to the uncertainty of using the Van de Graaff generator, a wand connected to a kilovolt power supply provided a known amount of charge to the ball. Instead of a disk rotating around, the single ball in this model was expected to receive an impulse of force when the current was shut off causing it to twist the torsion wire, providing the angle θ that the wire was twisted. Shut-off times were measured for two different solenoids, 715 and 3250 turns, using an oscilloscope to measure the amount of time it took for the current in the solenoid to reach 0 A after the DC power supply was shut off. This was crucial because to find the force on the charged ball in Eq. 16, the time rate of change of the magnetic field was needed. According to Eq. 16, a faster shut-off time would produce a larger force. The change in the magnetic field was found by using a gaussmeter to measure solenoid's magnetic field at the charged ball when current was running through the solenoid and subtracting the magnetic field when the solenoid was turned off.

RESULTS & ANALYSIS

For the physical rod model, it was necessary to test the shut-off times for two different solenoids, as discussed in the Physical Rod Model subsection of the Procedure. The shut-off time test measured the shut-off time of the solenoid with 715 turns for a current range of 0.463 to 1.091 A and a current range of 0.065 to 0.500 A for the solenoid with 3250 turns. The current range was lower for the solenoid with 3250 turns because increasing the current further overloaded the amount of voltage the DC power supply could provide. Figure 5 shows that the shut-off times for the 715 turn solenoid were less than

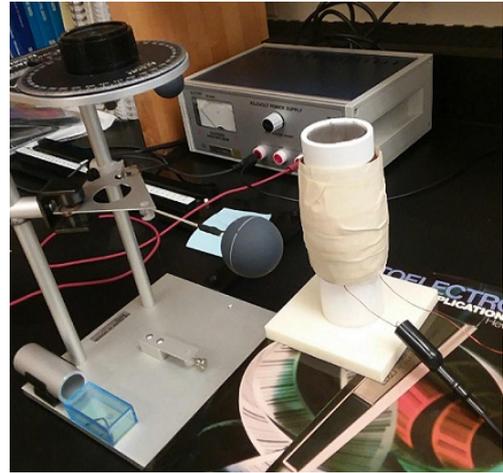


FIG. 4: Photograph of the physical rod model. The kilovolt power supply in the background was used to put a known charge on the dark gray hollow sphere in the foreground. The gray sphere was supported by a torsion wire. When the torsion wire twisted, the black knob on top of the apparatus was used to return the wire to equilibrium and determine the angle the sphere rotated.

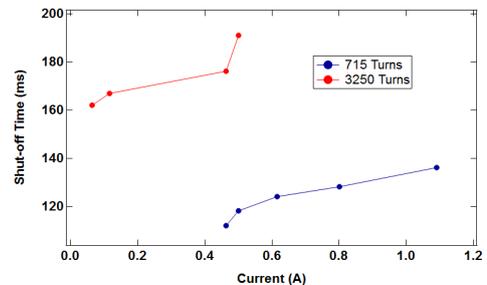


FIG. 5: Shut-off times vs. current for solenoids with 715 and 3250 turns. The shut-off times for the solenoid with 3250 turns is shown in red and the solenoid with 715 turns is shown in blue as various currents ran through the solenoids. The solenoid with 715 turns could handle a higher current and also had quicker shut-off times than the larger solenoid.

the shut-off times for the 3250 turn solenoid. The 715 turn solenoid shut off fastest at 0.463 A with a time of 112.0 ms. The 3250 turn solenoid shut off fastest at 0.065 A with a time of 162.0 ms.

During the planning process for this experiment, Dr. Lindner and I came up with theoretical values to test the equations in the Theory section. In the experimental results, however, many of these values were drastically different. Table I compares the theoretical values that were hypothesized to produce a visible movement of the metal sphere with the results measured using the physical rod model.

For the experimental values, I measured ΔB , Δt , R , and r , but I calculated Q using Eq. 13. Plugging these values into Eq. 9 provided a value for the torque on the sphere. Then, the force on the charged sphere was found

TABLE I: Theoretical and Experimental values for the variables measured in the physical rod model of the experiment.

Variable	Theoretical	Experimental
ΔB	0.1 T	0.0014 T
Δt	1.0 s	0.136 s
R	0.1 m	0.023 m
r	0.1 m	0.061 m
Q	1×10^{-5} C	1×10^{-8} C
τ	1.0×10^{-7} N.m	8.9×10^{-13} N.m
F	1×10^{-6} N	1.5×10^{-11} N
θ	0.73 degrees	1×10^{-5} degrees

using Eq. 15. Lastly, the angle θ at which the sphere rotated due to the circulating momentum field was found by dividing the force by the torsion constant k , as in Eq. 17. The experimental torque on the sphere was 10^6 smaller than the amount of torque that I predicted would be created. This is due mostly to the change in magnetic field being 100 times weaker than my predictions and the charge on the sphere being 1000 times weaker than my predictions. During the time provided, I was unable to acquire sources to provide a stronger initial magnetic field or a higher charge on the sphere. Options to increase the change in magnetic field by a factor of 100 include creating a solenoid with a quicker shut-off time, increasing the amount of turns of wire on the solenoid, using a soft iron core instead of a steel core in the solenoid, and using a power supply that could provide much more current to the solenoid. MRI machines used in hospitals have a magnetic field strength of around 1.5 T by using superconducting magnets and liquid helium [5]. Possible ways to increase the amount of charge on the sphere include using a power supply that can provide a higher voltage to the sphere or increasing the radius of the sphere. The theoretical angle that the sphere would rotate on the torsion wire was estimated to be 0.73° . However, the experimental results and calculations showed that the force on the sphere was causing it to move approximately 1/100,000 of a degree, which is impossible to see with the naked eye or to measure using the equipment I had available. Also, with an angle that small, air pressure changes and air drafts couldn't be ruled out for rotating the sphere.

Mathematica Modeling

When it became clear that I would not be able to acquire the needed resources to provide more magnetic field strength and charge on the sphere, I decided to create a model on Mathematica that would show the Feynman disk paradox using the design of the physical disk model. To do this, I first created two spheres using the *Graphics3D* function on Mathematica. These spheres

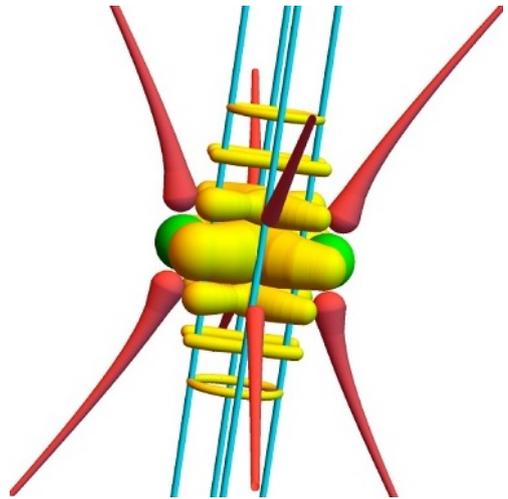


FIG. 6: Magnetic, electric and momentum field simulation. This Mathematica model shows the charges in green, the electric field lines of the charges in red, the magnetic field of the solenoid in blue, and the momentum field in yellow. Thicker lines represent a stronger field.

were used to represent the two charges in the physical disk model described in the Procedure section of this report. Following the creation of the charges, I used Mathematica to create red lines pointing in the direction of the electric field of the point charges, shown in Fig. 6. These electric field lines were thicker closer to the charges to show that the electric field grew weaker the further away from the charges it was measured. Next, the magnetic field lines were drawn. They consisted of straight blue cylinders in the place of where the inside of the solenoid was in the physical disk model, shown in Fig. refFront1 as well. The final part of my first three dimensional Mathematica model was the creation of the momentum field in yellow. Fig. 6 shows that the momentum field lines are strongest towards the middle of the solenoid and weaker toward the top and bottom of the solenoid.

This initial Mathematica model did not include showing the direction in which these fields were flowing. So, I created a second model using tubes with arrow heads on them to show the direction of the magnetic, electric, and momentum fields. In the second model, the color of the charges was changed from green to red to show that the electric field lines were associated with them. The second model, shown in Fig. 7, showed that the magnetic field was flowing from bottom to top in the solenoid while the electric field flowed away from the positive charges. The momentum field, seen from above in Fig. 7 rotated in a clockwise direction at the top and bottom of the solenoid, while the rest of the momentum field flowed in both clockwise and counter clockwise directions, canceling itself out everywhere else on the solenoid. This brought to light the idea that the angular momentum comes from the crossed electric and magnetic fields at

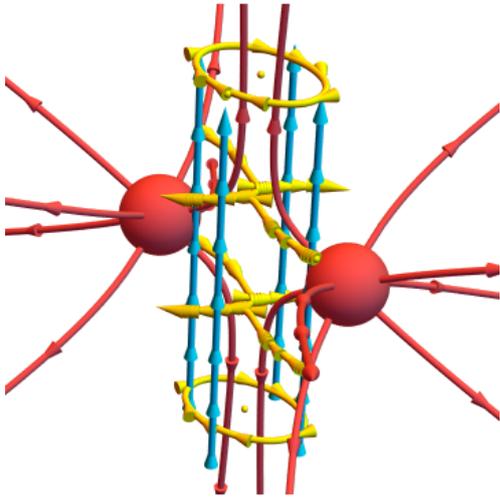


FIG. 7: Magnetic, electric and momentum field vector simulation. From above, this Mathematica model shows that the momentum field created by a change in the magnetic field, circulates clockwise at the top and bottom of the solenoid. In the center of the solenoid, the momentum field lines travel in opposite directions and cancel out.

the top and bottom of the solenoid, not the center. It is important to notice in Fig. 7 that both the top and bottom momentum fields travel in the same direction. If they traveled in opposite directions, the charges would not rotate as predicted in the Feynman disk paradox.

CONCLUSIONS

The goal of this experiment was to model the Feynman disk model both physically and virtually. I was able to create a model that represented all of the requirements described in Feynman's disk paradox. However, I found that I was unable to increase the magnetic field of the

solenoid and the charges on the spheres enough to create a visible or measurable (at least with my equipment) rotation due to the transfer of angular momentum. Using the measurements and data I collected, I was able to pinpoint which areas of the experiment could be improved and how. If the magnetic field of the solenoid could be increased by a factor of at least 100 by supplying more current to the solenoid and the charge on the sphere could be increased by a factor of at least 1000 by using a power supply with higher voltage and increasing the radius of the sphere, my calculations showed that the angle at which the charged sphere moved would be measurable using the torsion wire Coulomb balance used in this experiment. For the Mathematica modeling, in the future I would like to be able to manipulate the model to show what angle θ the spheres rotate when the magnetic field and charge on the spheres is varied.

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