An Investigation into the Impact of Particle Mass and Liquid Surface Tension on the Capillary Force Between a Floating Particle and a Nearby Massive Wall

Laura Grace

Physics Department, The College of Wooster, Wooster, Ohio 44691, USA

(Dated: May 6, 2015)

This experiment is an investigation into the factors that contribute to the force between a floating hydrophilic particle and a massive hydrophilic wall. The impact of particle mass and surface tension of the liquid were tested by video tracking the motion of toroidal cereal floating near the edge of a bowl of either tap water or soapy water. The graph of position versus time was fit with a high order polynomial to best fit the curve of the data while eliminating the noise in the sample. By differentiating polynomial fit of the position versus time graph of the cereal's motion towards the edge of the bowl, the acceleration of the cereal particle could be graphed against the sixth degree polynomial fit line of the position graph. From the relationships seen in these graphs, the acceleration due to the capillary force between a floating particle and a nearby wall was determined to be related to the surface tension of the liquid and the mass of the particle as $a \propto \gamma/m$, where γ is the surface tension of the liquid and m is the mass of the particle.

I. INTRODUCTION

An object floating on a liquid deforms the surface of the liquid in a meniscus surrounding the object. The meniscus effect or capillary effect results in an attractive force between two objects at an interface due to the interactions of their menisci. Leonardo da Vinci was the first person to record observations of the capillary force between water and another surface [1]. Many others continued to investigate the properties of liquid-solid interfaces. But it was not until 1805 that capillary action was described quantitatively by Thomas Young and Pierre-Simon Laplace. By 1830 Carl Friedrich Gauss found the boundary conditions responsible for the conditions at liquid-solid interfaces [1]. Such interfaces are very common in our world and an understanding of the forces in play at such locations can lead to some exciting applications. For example, it may be possible to assemble sheets of hydrophilic materials by allowing them to come together due to the capillary forces between individual particles [2]. This tendency of floating particles to clump together and drift toward the edges of the container due to capillary attraction has been named the "Cheerios Effect" after the everyday experience of cereal pieces clumping together as they float on top of milk in a bowl [3].

II. THEORY

A. Attraction of Particles due to the Capillary Effect

A particle floating on the surface of a liquid deforms the surface according to its chemical properties and mass. For example, the interface between water and a hydrophilic surface creates a positively curving meniscus, whereas a similar interface with a hydrophobic material creates a negatively curving meniscus as seen Fig. 1.

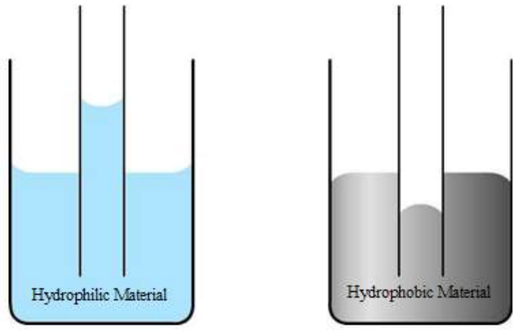

FIG. 1: The meniscus created between a hydrophilic material and water creates a positive meniscus as seen on the left. Between a hydrophobic material and water the meniscus created is negative, as seen on the right. In a hydrophilic capillary tube the meniscus raises the water level above the level outside the tube. A hydrophobic capillary tube would push the level of water in the tube below the outside level. Image from reference [5].

In this experiment all solids used were hydrophilic. The meniscus is formed by the capillary interactions between the liquid surface and the solid particle or wall. The capillary force comes from the wetting properties of the solid. The chemical properties of the solid make the liquid attracted to the solid, this causes the liquid to climb the wall in the shape of a meniscus. The liquid meniscus only climbs up the solid so far, though, as the capillary force is balanced out by the weight of the liquid. The angle between the liquid and the solid can be described by the expression $(\rho'' - \rho')gL = \gamma(1/r_1 + 1/r_2)$. Here $(\rho'' - \rho')gL$ describes the hydrostatic pressure at any point on the surface of the meniscus in terms of the density of the liquid, ρ, and the height above the free liquid surface, L.[4] In a tube of very small radius, the meniscus forms a hemisphere opening upwards, as seen in the hydrophilic material interface with water in Fig. 1, due to the capillary effects of the liquid. The meniscus

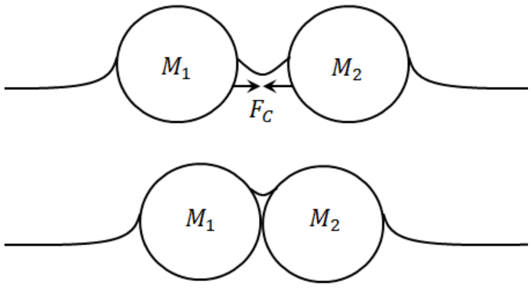

FIG. 2: The menisci surrounding two nearby particles combine to form a single meniscus between the particles, pulling them close together until the body of the particle prevents the meniscus from shortening further.

between the particle and the wall of the container pulls the particle closer to the wall so that the meniscus is as small as possible. The closer the particles are together the smaller the meniscus and the less distance the meniscus is stretched over. The shortening of the meniscus draws the particles together as seen in Fig. 2

Another way to think about the force of attraction between a particle and the meniscus of the container wall comes from the buoyant property of the particle. Since the particle is buoyant and floating on top of the water, it tends to move toward the highest possible point in the liquid [3]. Similar, but opposite to the idea of a ball rolling down a hill. This is due to the buoyant force from the water being greater than the weight of the object as shown in Fig. 3. Since the water bends up toward the hydrophilic wall, the particle can ascend to a slightly higher location by traveling up the meniscus. This gives the appearance of the particle being attracted to the wall itself.

B. Measuring the Surface Tension of the Liquid

The surface tension of a liquid can be measured by finding the rise of a liquid in a capillary tube of sufficiently small inner radius. The contact angle between water and many materials is well defined. In this experiment capillary tubes made of borosilicate glass were used. The contact angle between water-rinsed borosilicate glass and water is known to be $\theta \approxeq 25°$[6]. The amount a liquid rises in a capillary tube can be described by

$$\gamma = \frac{rhdg}{2cos\theta} \tag{1}$$

where γ is the surface tension of the liquid, r is the radius of the capillary tube, h is the rise height of the liquid in the capillary tube, d is the density of the liquid, g is the acceleration due to gravity (9.8 m/s^2) and θ is the contact angle between the liquid and the tube walls [7].

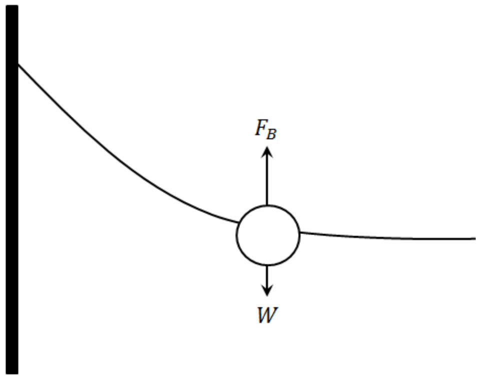

FIG. 3: The buoyant force, F_b, is greater than the weight, W, of the particle. This allows the particle to sit on the surface of the liquid. The particle can sit higher on the liquid surface if it moves left, up the meniscus, appearing to be attracted to the wall of the bowl.

Therefore if the radius of the capillary tube is known, the rise height inside the tube and the density of the liquid can be measured. From these values the surface tension of the tested liquid can be calculated.

III. PROCEDURE

A. Set Up

A Casio High-speed Exilim EX-FH20 camera was mounted above a round dish with a flat bottom and vertical sides as shown in Figure 4. The camera was angled so that the lens was pointed vertically down, directly above the dish. A ruler was placed next to the dish at approximately the height of the liquid in the bowl. This ruler would be used as a scale in the videos of the trial runs. As seen in Figure 5 a set of calipers was modified so that a toroidal cereal piece, in this experiment either Cheerios^{TM} or Fruit Loops^{TM}, hereafter referred to as cheerios or cereal, could be placed on a toothpick attached to one side of the calipers. The other side of the calipers was placed on the outside of the bowl edge. This setup allowed for the distance between the center of the cereal and the inside of the bowl edge to be set to a defined separation. The bowl was filled with 600 mL of either tap water or soapy water. Trials were run of various masses of cereal, where they were placed on the calipers at a set distance from the side of the bowl before being released to float freely on the surface of the liquid. At the end of the trials one sample of the liquid was separated out into a shallow glass dish to measure the surface tension. Another sample was separated into a graduated cylinder and massed to find the density of the liquid.

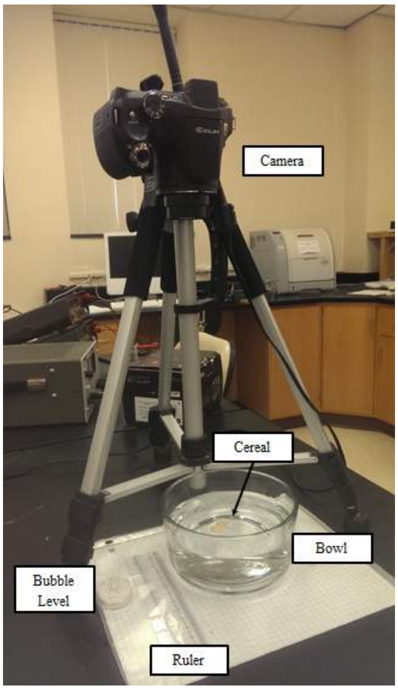

FIG. 4: A Casio High-speed camera was placed above the bowl so that the video was taken from directly above the bowl. The bowl was placed on graph paper so that the deformation of the surface of the water could be seen from above, as it distorted the image of the gridlines.

FIG. 5: A set of calipers were modified with a toothpick so that the toothpick could be placed in the open center of the cheerio. This insured that the cereal started at rest before drifting toward the side of the bowl. The clip with toothpicks attached, next to the calipers in the figure, held the cereal on the modified calipers until the calipers were in place to hold the cereal at the desired distance from the edge of the bowl.

B. Taking Data

To actually track the motion of the cereal on a liquid the camera was put on video and pointed straight down at the top of the bowl. A cheerio was placed on the toothpick on the modified calipers shown in Fig. 5. The cereal was held onto the toothpick using the clip seen next to the modified calipers in Fig. 5. When the tip of the toothpick touched the surface of the liquid the cheerio was allowed to rest on the surface of the liquid. The other side of the calipers was pressed against the outside edge of the bowl. This allowed the cheerio to start at rest from a determined distance away from the edge of the bowl.The calipers were slid up the side of the bowl, releasing the cheerio from the toothpick to float freely on the liquid. The camera recorded the view of the bowl from the time the cheerio was placed on the calipers to the time it came to rest on the side of the bowl. This process was repeated for each of the massed cheerios and fuitloops. The amount of water in the bowl was assumed to be great enough that the addition of the sugar from the cereal was a minimal contribution to the makeup of the liquid for the following trials. After the trials for a group of massed cheerios and fruitloops, a sample of the water was separated into a shallow dish to measure the surface tension and another sample was massed to determine the density of the liquid. The bowl was rinsed with water and dried with paper towel before the next trial. The videos of the different cereal particles were uploaded to a computer and analyzed using Tracker, a video tracking software.

C. Measuring the Surface Tension

The sample of water separated into the shallow dish was placed on a laboratory jack so that the surface of the water was level with the lens of the camera. The camera was placed on a tripod on the floor and adjusted so that the camera was at the same height as the water in the dish. Slight adjustments in height were made by moving the height of the jack as it was more precise than the adjustments on the tripod. A capillary tube was placed in the dish so that it was perpendicular to the surface of the water in front of the camera. Image 6 shows how the capillary tube was held in place in front of the camera. Two bottles of equal height held a protractor above the dish. The protractor has a small hole in the flat edge, which was aligned so that the the capillary tube placed through the hole entered the liquid at the edge of the dish in front of the camera lens. This allowed for the clearest picture to be taken of the amount the liquid rose in the capillary tube to most accurately measure the surface tension of the liquid. The image of the capillary tube was uploaded to a computer, where the image was analyzed using Tracker, a free video and image analysis software. One image was taken of a capillary tube lying next to a ruler on the same plane. Using the ruler placed next to the capillary tube for a reference scale, Tracker was able to calibrate the scale of the image and determine the distance between two markings on the capillary tubes, which were actually micro-pipettes, marked at every microliter of volume. This distance was measured to be 1.769 cm to an accuracy of ± 0.003 cm, accounting for a slight amount of parallax which may have occurred if the ruler were not exactly on the same plane as the capillary or if the measurements were not exactly marked in the image. The images of the rise of

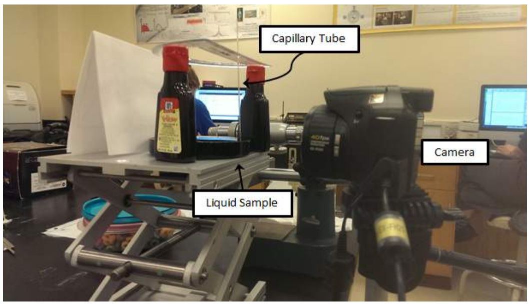

FIG. 6: Set up of camera, capillary tube, and liquid sample to measure the surface tension of the liquid. The camera is set up so that the lens points directly at the surface of the liquid in the dish. The distance the liquid rises in the capillary tube is related to the surface tension of the liquid.

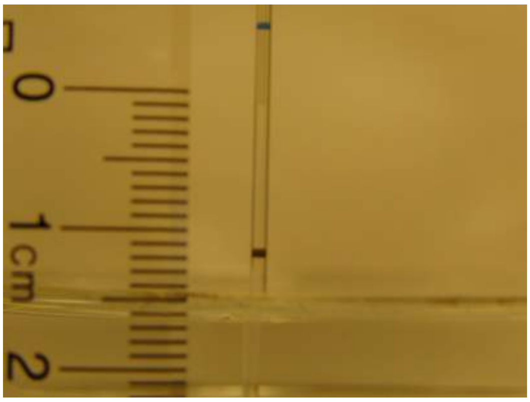

FIG. 7: Picture of the rise of the liquid in a capillary tube from the side. The distance between stripes on the tube was measured to be 1.769 $\pm$ 0.03 cm. This value was used to calibrate Tracker to measure the rise in the capillary tube in the picture.

the liquid in the capillary tube were then analyzed in Tracker, using the markings on the tube as the distance calibration. The height of the liquid in the tube was measured using a "measuring tape" feature of Tracker. Using the distance calibration in the image Tracker can measure the distance between two points in the image. Choosing the top of the liquid in the dish and the top of the liquid inside the capillary tube along the length of the tube, Tracker could determine the rise height of the liquid in the capillary tube. This height could be used, along with the density of the liquid, to find the surface tension of the liquid according to Eqn. 1.

D. Finding the Density of the Liquid

Density is simply the mass of an object per unit volume. A 10 mL graduated cylinder was placed on a scale accurate up to $\pm$0.01 g. The scale was then zeroed and 10 mL of the trial liquid poured into the graduated cylinder, which was accurate to $\pm$0.1 mL. The scale measured the mass of the added liquid. The mass divided by the volume of liquid in the cylinder gives the density of the liquid. The surface tension of the liquid could then be found using Eqn. 1.

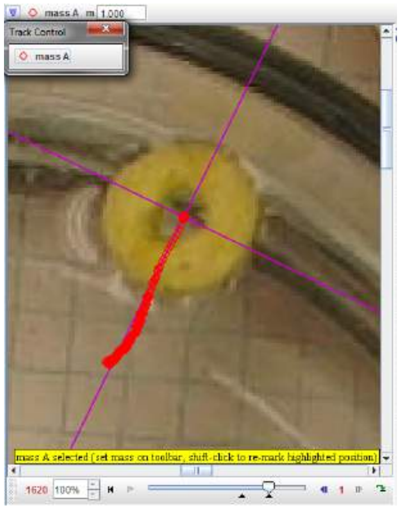

FIG. 8: An example of a video track for a fruit loop in water. The red trail is the motion of the center of mass over the selected frame range. The purple lines are the coordinate axes. The origin is set to the center of the cereal when it is at rest against the wall of the bowl to normalize the tracks for different size cereals.

IV. RESULTS AND ANALYSIS

A. Video Tracking

The videos from the trials were loaded into Tracker, a video and photo analysis software developed for physics education through videos. Tracker can calibrate the distance scale for a video given a reference length. Using the ruler placed at approximately the same height as the surface of the water, Tracker was calibrated to output distances in centimeters. The video was then cropped to the frames where the cereal went from rest at the distance determined by the calipers to rest at the edge of the bowl. The motion of the cereal was tracked frame by frame as a point mass located at the center of the open middle of the cereal. This normalized the data to account for the different sizes of cereal, cheerios and fruit loops. The coordinate origin was set as the location where the point mass came to rest at the edge of the bowl, so that the cereal accelerated toward the origin. The motion of the cereal was plotted with polar coordinates where r was the distance from the rest location at the edge of the bowl. These r values, along with the corresponding times in seconds, were imported into Igor Pro for further analysis.

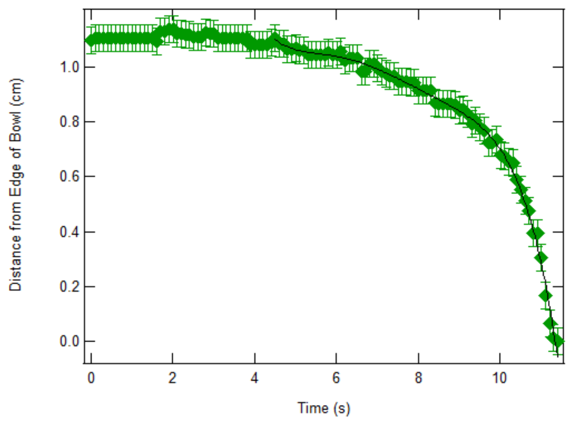

FIG. 9: The graph of position versus time for a fruit loop in soapy water. The data are shown as green diamonds, the sixth order polynomial fit line is shown in black.

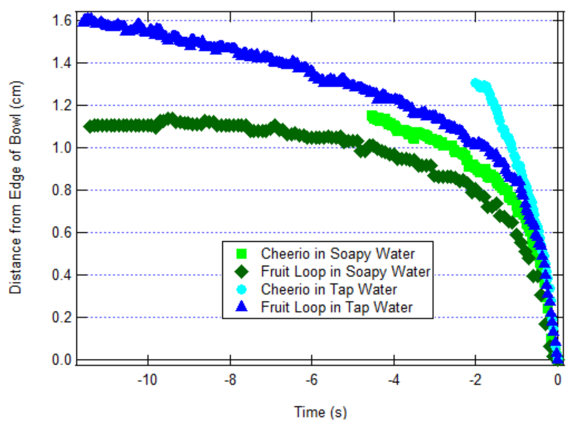

FIG. 10: The plot of position versus time for two different sizes of cereal in two liquids with different surface tensions. From left to right: a fruit loop in soapy water, a cheerio in soapy water, a fruit loop in tap water and a cheerio in tap water.

B. Finding the Acceleration of the Cereal

The position and time data imported from Tracker into Igor Pro were graphed against each other for the best trial of each of the data conditions. The data were then fit with a high order polynomial in the region where the motion of the cereal became noticeable, or where the data begin to slope downward. Using the polynomial fit of the data helped to decrease the noise in the sample while maintaining the shape of the data, an example is shown in Fig. 9. The graph of location versus time for all four data conditions is shown in Fig. 10. The light lines are the data for the cheerios, and the dark lines are the data for the larger fruit loops. The blue color designates that tap water was used for the liquid. The green color denotes the soapy water, which lowered the surface tension from that of the tap water. To allow for easy comparison of the various data conditions, the data curves were adjusted

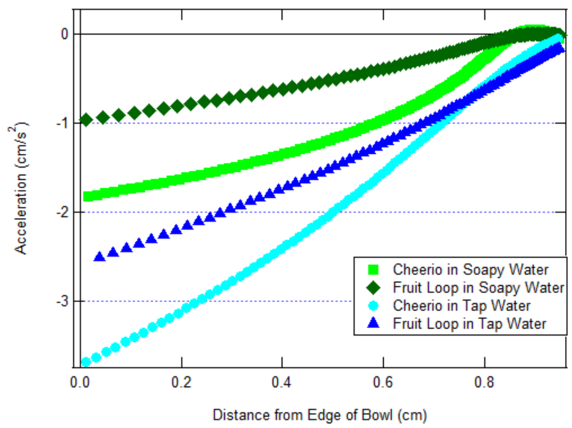

FIG. 11: The graphs of the differentiated accelerations versus the polynomial fit line of the data runs in Fig. 10. From top to bottom: a fruit loop in soapy water, a cheerio in soapy water, a fruit loop in tap water and a cheerio in tap water.

along the time axis so that the origin represented the point where the cereal came to rest at the edge of the bowl. Therefore the time read as negative along the x-axis, or time before the cereal came to rest. This graph shows that for a lower surface tension (soapy water) the cereal had to be closer to the edge of the bowl to feel the effects of the meniscus force from the wall. It also shows that the heavier fruit loops had to be closer to the edge of the bowl than the lighter cheerios for the same liquid to respond to the capillary force.

The polynomial fit line for each of the trials was then differentiated. This new curve was differentiated again. The second differential of the position versus time fit was plotted against the polynomial fit of the position versus time curve. This final curve is shown in Fig. 11 and effectively gives the acceleration of the cereal versus the position. The data was cropped to the areas where the acceleration was changing, at farther distances the acceleration was close to zero. This graph shows that the acceleration increases as the distance to the edge of the bowl decreases. For cereal particles of the same size, decreasing the surface tension decreased the acceleration by nearly half. Increasing the size of the particle also decreased the acceleration. This is likely because with the greater mass of the fruit loop, there was also more surface area of the cereal in the water. This would increase the frictional drag.

The graph of acceleration, therefore, gives the relationship of

$$a \propto \frac{\gamma}{m} \tag{2}$$

where a is the acceleration of a hydrophilic floating particle near a hydrophilic wall due to the capillary effect, γ is the surface tension of the liquid and m is the mass of the particle. The force on a hydrophilic particle floating on a liquid near a hydrophilic wall is related to the surface tension of the liquid and inversely related to the mass of the particle.

V. CONCLUSIONS

This experiment was an investigation into the force between a particle floating on a liquid and a nearby hydrophilic wall. By video tracking the motion of a cheerio or fruit loop as it was released from rest near the wall of a bowl, the radius of the cereal from the final rest position on the wall of the bowl could be graphed against time. This was done for four different data conditions, heavy and light particles on liquids of high and low surface tensions. Specifically fruit loops and cheerios on tap water and soapy water. Since the video tracker was accurate up to ± 0.05 cm and ± 0.005 s there was little error in the analysis of the videos. Some error may have occurred in the data collection process if the cereal was bumped as it was released from rest on the modified calipers. Error may have also occurred if the cereal sat in the water or was submerged as it was placed on the surface of the liquid. If the cereal absorbed extra water in either of these ways, the mass would have increased and the particle would have sat lower in the water, increasing the frictional drag from the liquid. Differences in the sugary coating of the cereal may have allowed the cereal to soak up extra liquid, increasing the drag as well. To eliminate these sources of error further experiments might use a uniform particle, such as a plastic sphere, and change the mass without changing the surface area or shape. It would also be interesting to be able to measure the surface roughness of the particle to see if that factor influences the capillary force with the wall. Testing only the mass of the floating particle and the surface tension of the liquid; the observed impact on the acceleration between the particle and the wall gives the relationship $a \propto \gamma/m$, where γ is the surface tension of the liquid and m is the mass of the particle.

[1] "Capillary Action." Wikipedia, accessed April 22, 2015, http://en.wikipedia.org/wiki/Capillary_action.

[2] N. Xue, S. Wu, S. Sun, D. Quere, and Q. Zheng, Langmuir. 30, 49, 14712 (2014)

[3] D. Vella and L. Mahadevan, Am. J. Phys. 73, 9, 817 (2005)

[4] R.J. Hunter, *Foundations of Colloid Science*, 2nd ed. (Oxford University Press Inc., New York, 2001)

[5] Capillary tube image, accessed April 21, 2015, http://spaceflight.esa.int/impress/text/education/Images/Glossary/GlossaryImage014.jpg

[6] A. Sumner, E. Menke, Y. Dubowski, J. Newberg, R. Penner, J. Hemminger, L. Wingen, T. Brauers, and B. Finlayson-Pitts, Phys. Chem. Chem. Phys. **6**, 604 (2004)

[7] Wyklad, "Methods of Surface Tension Measurement" Maria Curie Sklodowska University, accessed March 24, 2015, http://zzm.umcs.lublin.pl/Wyklad/FGF-Ang/2A.F.G.F.%20Surface%20tension.pdf