Using a Light Bulb Filament to Verify Power-Temperature Proportionality in an Approximate Blackbody Radiator

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(Dated: December 12, 2019)

A perfect blackbody radiator can absorb all wavelengths of light and will re-emit this same amount of energy to remain in thermal equilibrium. Stefan and Boltzmann were able to discover and derive a proportionality between Power and Temperature to the 4th power, which will be verified within this report. Current and voltage were measured both at room temperature and high temperatures, both data sets giving respective resistances. By forming a ratio between a reference room temperature resistance and the high temperature resistances, a power-temperature proportional relationship was fit graphically to find the power term of temperature to be 3.81±0.03, within 5% of the accepted value of T^4. Error was propagated throughout all calculations to account for the uncertainty of manually taking measurements, especially as the reference resistance measurements were fluctuating on the meters because of their small values.

I. INTRODUCTION

In the late 1800s, after Hertz verified the existence of electric waves, thermal and visible electromagnetic waves became of growing focus [1]. Blackbody radiation was theorized by Wien as a black cavity with perfectly absorbing walls that could then radiate all absorbed wavelength of light [1, 2]. If the temperature of this perfect absorber is fixed and high enough, it must become a perfect emitter to stay in thermal equilibrium [1, 2]. Josef Stefan took these ideas and empirically discovered a proportion between the power emitted by black bodies and their absolute temperature to the fourth [3]. Boltzmann, a student of Stefan, wanted to solve the conflict between the second law of thermodynamics (a thermodynamic process must increase entropy and decrease free energy within a system [4]) and the concept of blackbody radiation. He used Hertz's findings along with Maxwell's equations to begin to prove the Stefan power-temperature relation. [1, 3].
This experiment aims to verify the concept of blackbody radiation by using the filament of a tungsten incandescent light bulb. While there will never be a perfect blackbody radiator on earth, the light bulb filament is a good approximation because its emits all visible wavelengths when the filament reaches high temperatures. Very little reflection takes place within the system because the filament emits light intrinsically because it is hot. To find these white light emmission temperatures, current and voltage were measured at both room temperature (when there was no visible emission) and high temperature (when the emitted light was white.).

II. THEORY

For an object to be defined as a blackbody radiator, it must be able to absorb all incident light with no dependence on the the angle of incidence, polarization, or wavelength of the incident light [5]. This object will be in thermal equilibrium; therefore, when it absorbs this incident light, it must emit the same amount of light to remain in thermal equilibrium. This light cannot be reflected off the surface for the object, it must be emitted intrinsically through the temperature of the object.
As objects increase in temperature, they begin to emit different wavelengths of light, emitting smaller wavelengths when the object has a higher temperature [2]. When some objects get hot enough, it will release all the wavelengths in the visible spectrum [2]. This is why at small currents, the filament appears to glow red, it is only hot enough to emit the larger red wavelengths. When the current is high enough, the filament becomes hot enough to glow white because it is now releasing the same amount of light for each visible wavelength.
These wavelengths of light must be emitted in packages, or quanta, from the radiators. All materials are capable of releasing electromagnetic radiation because all particles are always vibrating [6]. Since all photons contain a set amount of energy, the number of quanta released depends on this vibration, which will changed based on the temperature of the object [6]. Classical wave theory predicts that if an object can release higher and higher frequency photons, the energy density (dU/df, electromagnetic energy over a range of frequency) will diverge to infinity [6]. Experimentation proved this theory to be incorrect, as experiments show energy density reaching a maximum and then approaching zero as frequency diverged to infinity [6], which can be seen in Figure 1. This created the ultra-violet catastrophe, as the theory and the experimental results agreed in the smaller wavelengths, but diverged from one another as the the frequency range went into ultraviolet [6]. Classical wave theory shows a function for spectral energy over a small range of frequencies,

$$\frac{du}{df} = k_b T \times \frac{8\pi V}{c^3} f^2 \tag{1}$$

where k_b is the Boltzmann constant, V is the volume of the radiator, and c is the speed of light. However, this energy density function does not align with experimental results. The problem with this assumption is that it takes

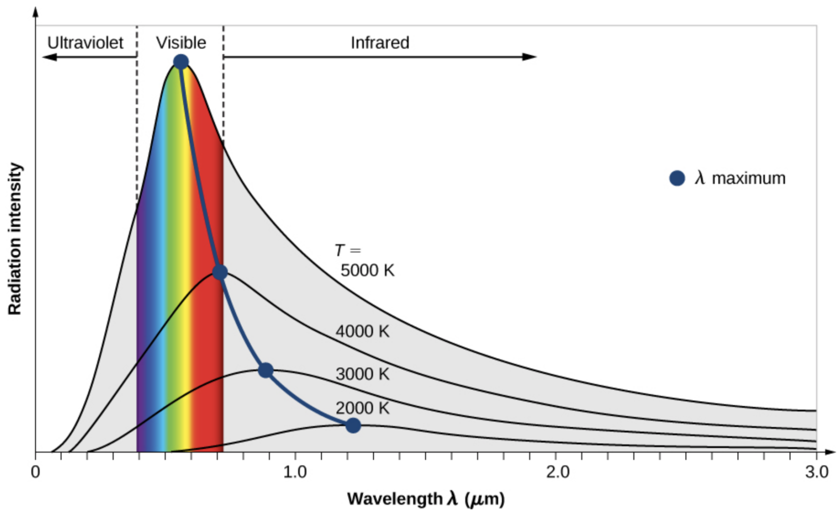

Figure 1: The experimental results for the relationship between temperature and radiation energy. This plot proved the theory to be incorrect because intensity does not diverge to infinity, but instead reaches a maximum in the visible spectrum. This figure was taken from the Openstax University Physics III- Optics and Modern Physics textbook [2]

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light to only act as a wave. Planck used the assumption that light acts as a particle in this relationship where the energy

$$E = nhf \tag{2}$$

is equal to the frequency of the light, multiplied by the constant, h. Planck could then calculate the number of photons, n. When Planck replaced the k_BT in the energy density equation with the energy of a photon considered as a particle as

$$\frac{du}{df} = \frac{hf}{e^{\frac{hf}{k_BT}} - 1} \times \frac{8\pi V}{c^3} f^2 \tag{3}$$

he was then able to match the results of the experimental curves seen in Figure 1 [6]. This equation must then be integrated over for all possible values of frequency to account for all wavelengths of light

$$u(f) = \int_0^\infty \frac{8\pi V h f^3}{c^3(e^{\frac{hf}{k_BT}} - 1)} df \tag{4}$$

by combining Eq. 3 into one integration term [7]. The volume term was also divided out so the integral produces the energy per unit volume, represented now by U. Taking out constants out of the integral, U is now

$$U = \frac{8\pi h}{c^3} \int_0^\infty \frac{f^3}{(e^{\frac{hf}{k_BT}} - 1)} dx. \tag{5}$$

A substitution can then be made for f and df to turn them into a general variable x. This takes the form [8]

$$x = \frac{hf}{k_BT}. \tag{6}$$

Taking the derivative of this equation returns

$$dx = \frac{h}{k_BT} df \tag{7}$$

and we can then insert this derivative into the integral. An easier integral is found to be

$$U = \frac{8\pi(k_BT)^4}{(hc)^3} \int_0^\infty \frac{x^3}{(e^x - 1)} df \tag{8}$$

that results in the total energy per volume. The result of the integral is $\pi^4/15$. Inserting that into the expression for U, U is now

$$U = \frac{8\pi^5(k_BT)^4}{15(hc)^3} \tag{9}$$

and we can then redefine this equation to be

$$U = \sigma T^4 \tag{10}$$

where σ is the Stefan-Boltzmann constant, which is equal to $\sigma = 8\pi^5 k_B^4/15(hc)^3$. This derivation proves the relationship between total energy density (U) and the temperature of the radiator. Since we will be using an imperfect blackbody, an emissivity factor ϵ needs to be added in[3]

$$U = \epsilon\sigma T^4 \tag{11}$$

where the maximum value of ϵ is 1 if it is a perfect blackbody.

Since we will be using a light bulb in a circuit for this experiment, I will be able to easily measure the current and voltage of the system and then calculate power. It would be helpful to have a proportional relationship between power and temperature. We can find this through utilizing the relationship between radiated flux F and energy density [5],

$$F = \frac{c}{4}U \tag{12}$$

before U is integrated. Because flux is dimensionally [5]

$$F = \frac{energy}{area \cdot time} \tag{13}$$

it is also proportional to power, the parameter we are looking to relate to temperature

$$F = \frac{Power}{Area}. \tag{14}$$

This fractional term can then be inserted into Eq. 12 , and Eq. 11 can also be inserted for U,

$$\frac{P}{A} = \frac{c}{4}\epsilon\sigma T^4 \tag{15}$$

creating a proportionally between temperature and power. To make this equation simpler, σ can be multiplied by c/4,

$$\sigma = \frac{2\pi^5 (k_B T)^4}{15 h^3 c^2} \tag{16}$$

to redefine the value of σ. Solving Eq. 15 for power with the redefined σ,

$$P = \epsilon\sigma A T^4 \tag{17}$$

a simple equation equating power to be proportional to temperature can easily be found.

III. PROCEDURE

In an attempt to mimic an almost perfect blackbody radiator here on earth, the apparatus used a tungsten incandescent light bulb connected in series to a power source and ammeter, and connected in parallel with a volt meter, as seen in Figure 2. In order to calculate the absolute temperature of the filament, a room temperature reference resistance was needed to compare to the resistances to be measured at higher temperatures. A set of currents and voltages were measured between -40 and 20 mA, when the light bulb was not hot enough to emit any visible light. These numbers were more difficult to

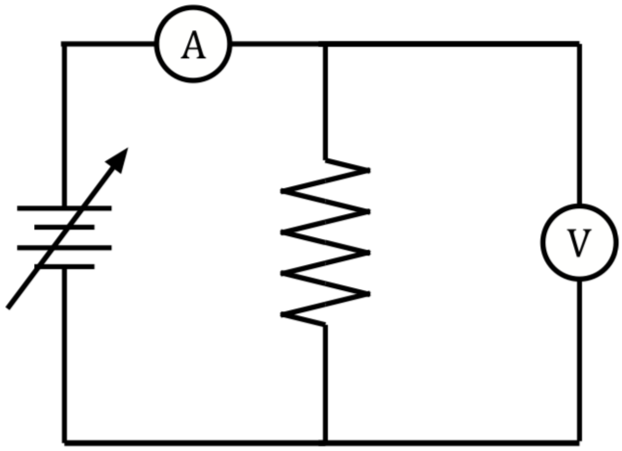

Figure 2: Diagram of the apparatus used for the experiment. The resistor in the middle of the circuit represents the light bulb. This figure was taken from the Junior IS Lab Manual [3].

measure as the reading fluctuated on the ammeter and voltmeter, most likely because the values were so small. A standard 10 mV uncertainty would be significant to this data set because the uncertainty is the same magnitude as the actual values of the measurements.

A plot of voltage vs current (Figure 3) was created to be able to tell the resistance when the bulb remained at room temperature (taken to be about 300 K). Because the slope of this line needed to be accurate, the negative values of voltages and currents, resulting from the offset in the instruments, were used to provide more data points without heating up the filament. Similarly, increasing the number of data points with these negative values helped to counteract the error in recording the fluctuating values from the meters.

I then applied significantly more current to the light bulb, between 2-3 V. Since the filament has such a low resistance, this caused the temperature of the filament to heat up quickly. The light bulb became hot enough to glow white. The filament was not reflecting light, but the temperature of the filament was high enough to intrinsically emit the same magnitude of all visible wavelengths. A range of current and voltage data points were taken in this range of high filament temperature (2-3 V). Then resistances for these high current and voltage values could be calculated. These values were less fluctuating and could be recorded with more certainty out to three significant figures, most likely because of the higher values of both current and voltage compared to the room temperature data set. The standard 10 mV uncertainty was less significant within these values because the uncertainty is three orders of magnitude smaller than the measurement values. Using these measurements, we could find the resistance at high temperatures. These resistances could be used to create a ratio with the reference resistance. The

Pasco reference sheet gave a graph of resistance ratios vs temperature. I fit this data set in Igor to find a general function in which to insert my recorded resistance ratios.

IV. RESULTS AND ANALYSIS

A. Reference Resistance

First a reference resistance was calculated to be compare the the resistances at higher temperatures. This was done by measuring a data set of current and voltages. By Ohm's law,

$$V = IR \tag{18}$$

resistance can be calculated by dividing the voltage by current. Recall that I kept the filament as close to room temperature as possible to get an accurate reference resistance. A plot was created whose slope would be equal to the reference resistance. Figure 3 shows this relationship. The slope of this plot is $0.3256 \pm 0.0002\ \Omega$. From this point on, this value will be taken as the reference resistance at room temperature. It will be compared to other resistances when the filament temperature was increased above room temperature.

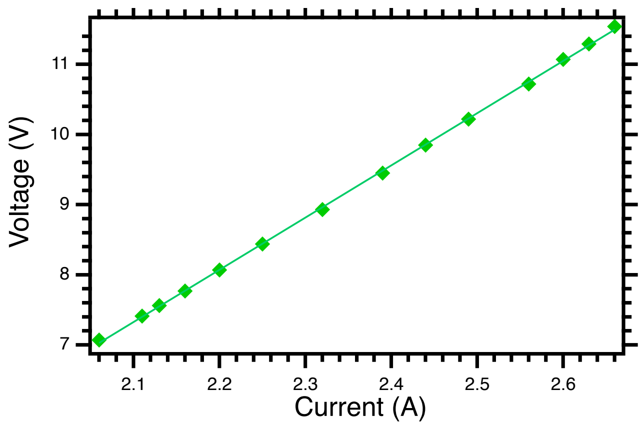

Figure 3: Measured voltage and current for the light bulb at room temperature, showing the Ohm's law relationship between voltage vs current. The slope of this graph, $0.3256 \pm 0.0002\ \Omega$, is the reference resistance because it was calculated when the filament was not hot enough to emit visible light.

B. Blackbody Temperatures

The Pasco reference sheet provided a chart of ratios of resistance as a function of temperature to the reference resistance. The reference resistance needs to be a very accurate number to avoid larger errors in values calculated with this value, so negative data points produced by the offset of the meters were used. This function allowed me to calculate temperature for any ratio of R/Rref.The ratios corresponded to a certain temperature of the filament. I used this information to find a function for this relationship (R/R_{ref} vs T) to be able to apply my resistance ratios to and find the temperature of the filament at these different points. This theoretical information is displayed in Figure 4. When applying a curve fit to Figure 4, I found a power law to be the most accurate,

$$T = 11.8 + 285.04\left(\frac{R}{R_{ref}}\right)^{0.84183} \tag{19}$$

specifically isolating T on the y axis to be able to insert my recorded ratios easily. Values within this functions are reported to many significant figures because the added uncertainty of calculating individual points with this function will not be significant compared to the uncertainty found from the original current and voltage measurements. Because of this, I chose to keep all significant figures to avoid rounding error, adding further uncertainty. All calculations with this function were performed with all significant figures seen in Eq. 19. Through Eq. 19, I obtained temperature for all the measured ratios. I then calculated the power from the large voltage data points

$$P = I^2R = VI \tag{20}$$

to be able to relate the temperatures to the power emitted. To more clearly see the relationship my recorded data showed between power and temperature, I took the logarithm of both sides so the slope of this straight line

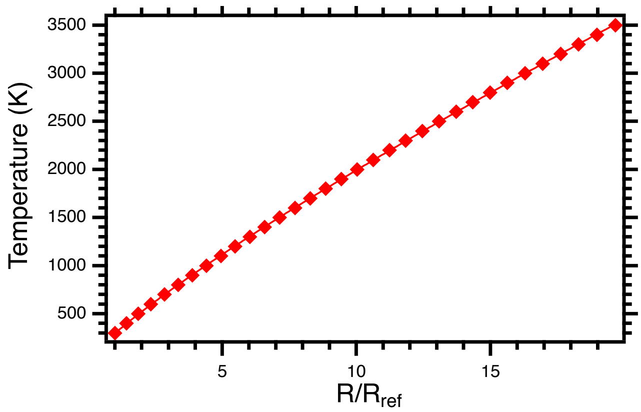

Figure 4: A graph using the theoretical resistance divided by reference resistance vs temperature to be able to obtain a function to plug in the measured resistance ratios. Igor fit this function to be a power law with a power of 0.84143.

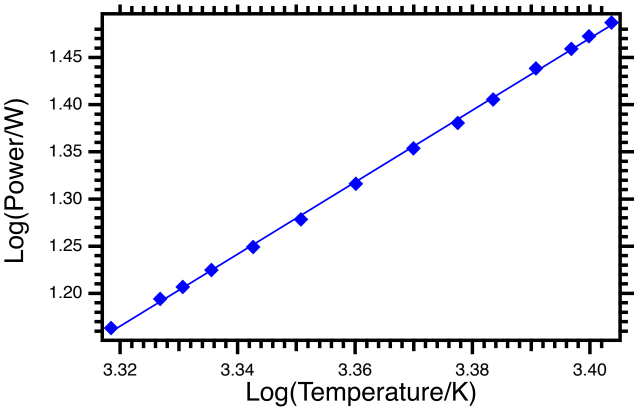

Figure 5: A plot to show the proportional relationship between energy and temperature. The logarithm of both values was taken so the power of temperature would be the slope of the linearized graph. The slope of this graph was 3.81 ± 0.03.

produced would be the proportion of power to temperature. This relationship is seen in Figure 5. The slope of this graph was 3.81 ± 0.03, within 4.6% of the power term of the expected T^4.

C. Error Propagation

To keep the reporting values as accurate as possible and to know how certain the measured and calculated number were, error propagation was used within all calculations. Since all calculations were simple multiplication or division, a simple sum of squares could be used [3]. This process was done when calculating the reference resistance, high current resistances, the ratio of the high current resistances to the reference resistances, and power. I estimated the manual current and voltage measurements to be confidently recorded out to two significant figures with $\pm 1\%$ uncertainty. The values for the T vs R/R_{ref} (seen in Figure 4) fitting function (Eq. 19) used to calculate the temperatures had numerical uncertainty in its coefficients, but this became negligible when data points with more significant uncertainty were inserted into it. Because the uncertainty within the measurements was so much greater than the uncertainty within the function applied to the measurements, propagating the error was focused within the calculations directly using the manual current and voltage measurements.

V. CONCLUSIONS

From these results, the proportionality of power to temperature to the 4th power could be verified within 5%. While the measured value, 3.81 ± 0.03 does not have a large enough uncertainty to include the expected value of 4, given that we take other forms of energy loss (condition, convection) to be negligible and just using electrical power in as approximately equal to the radiated power out, the 5% difference in values is still considered reasonable. When an object of high resistance is given high amounts of current, it will heat up and begin to emit light of increasingly small wavelengths. This was a significant discovery that lead to many things in both the fields of thermal physics and quantum mechanics.

[1] T. S. Kuhn. Black body theory and the quantum discontinuity, 1894-1912. *Philosophy of Science*, 49(2):295–297, 1978.

[2] Bill Moebs Samuel J. Ling, Jeff Sanny. University physics III- optics and modern physics (openstax). `https://phys.libretexts.org/Bookshelves/University_Physics/Book-3A_University_Physics_(OpenStax)/Map-3A_University_Physics_III_-_Optics_and_Modern_Physics_(OpenStax)/6-3A_Photons_and_Matter_Waves/6.1-3A_Blackbody_Radiation`. Accessed: 2019-07-18.

[3] Department of Physics, The College of Wooster, Wooster, OH. *Junior IS Lab Manual*, 2019.

[4] Piotr Ćwikliński, Michał Studziński, Michał Horodecki, and Jonathan Oppenheim. Limitations on the evolution of quantum coherences: Towards fully quantum second laws of thermodynamics. *Phys. Rev. Lett.*, 115:210403, Nov 2015.

[5] Joanne C. Zwinkles. *Encyclopedia of Color Science and Technology*, chapter Blackbody and Blackbody Radiation, pages 1–5. Springer Berlin Heidelberg, Berlin, Heidelberg, 2014.

[6] Randy Harris. *Modern Physics*. Pearson Addison-Wesley, 2008.

[7] Claes Johnson. *Mathematical Physics of BlackBody Radiation*. Icarus iDucation, 2012.

[8] J. S. Tenn. Planck's derivation of the energy density of blackbody radiation. `http://web.phys.ntnu.no/~stovneng/TFY4165_2013/BlackbodyRadiation.pdf`, 2007.