The Effects of Fluid in a Can Rolling Down an Incline

Jon Rosch
Physics Department, The College of Wooster, Wooster, Ohio 44691, USA
(Dated: May 9, 2006)

An aluminum can containing variable amounts of fluid was rolled down an incline. A decrease in the acceleration of the can was observed when approximately 10 mL of fluid were present. This is theorized to be due to a torque from a droplet acting opposite to the direction of rolling motion of the can. Oscillatory motion was observed when between 30 and 80 mL of fluid were present. This was believed to be due to fluid traversing back and forth within the can, although no direct observations were made. The rotational inertia of the can with fluid as a function of its total mass was determined to not be a linear function.

INTRODUCTION AND THEORY

The motion of a cylindrical body down an incline is a classical problem encountered in introductory physics. The mass distribution about an axis of rotation yields the body's rotational inertia. The acceleration of a rigid, cylindrical body rolling without slipping down an incline is inversely proportional to its rotational inertia.

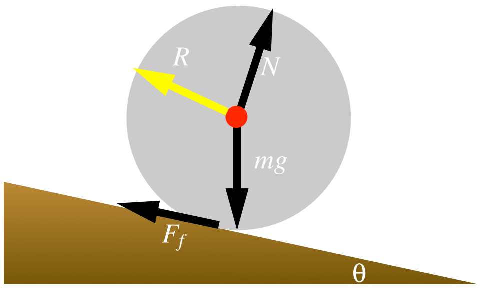

FIG. 1: Cylinder rolling without slipping [1].

Using the plane as the coordinate of motion and the direction pointing towards the bottom of the inline as positive, one can use Newton's second law to find

$$Mg\sin\theta - F_f = Ma \tag{1}$$

where M is the total mass of the body, F_f is the frictional force, and a is the acceleration of the center of mass. Knowing that torque acts on the cylinder via the frictional force F_f at a radius R from the center of mass, the rotational inertia of the rigid body can solved as

$$I = \frac{MR^2(g\sin\theta - a)}{a} \tag{2}$$

This implies that the rotational inertia can be determined for a body of known radius, mass, and acceleration with an inclination angle of θ. With the addition of fluid to such a system, it is no longer a rigid body and abnormalities in the acceleration occur. Determination of the rotational inertia for all objects will be done by performing linear fits ($y = a + bx$) to velocity vs. time plots and using the slope to represent the acceleration ($\dot{\mathbf{v}} = \mathbf{a}$) in equation 2.

To study the effect of various amounts of fluid in a can on the rotational inertia of the system, water was added to an aluminum can which was then allowed to roll down an incline. The can's velocity would then be measured and the rotational inertia would be determined. A plot of the rotational inertia as a function of fluid mass was constructed as well as a plot of the acceleration as a function of the mass of fluid. The latter will be of particular interest when developing a model for this experiment.

EXPERIMENTAL SETUP AND PROCEDURE

In order to collect data, a Macintosh Computer running OS X 10.3.9 was used in conjunction with a Science Workshop 750 Interface (USB Interface with Data Studio 1.9.7r8) and a PASCO Motion Sensor II. Figure 2 depicts the testing apparatus.

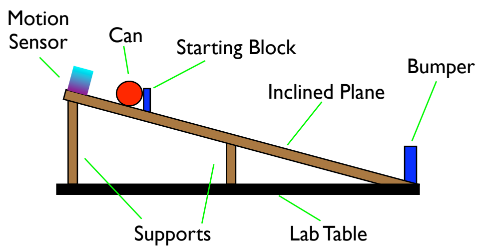

FIG. 2: Experimental Setup.

The supports were two adjustable jacks to minimize bending of the surface as the can was rolling. A bumper was placed at the end of the incline to reduce the likeliness of damage to the can. The starting block was used to make the can perpendicular to the edge of the incline to ensure a straight path down the incline. The motion sensor was set to a sampling rate of 40 Hz and programmed to start data collection once a velocity of $0.1\,\mathrm{ms}^{-1}$ was measured and to stop once a position of 2 m was measured (since this was the limit of the motion sensor's observable range). The wooden incline was sanded to obtain a smooth surface and to possibly reduce error. Duct tape was used to seal the can (its effect will be considered).

ANALYSIS AND DISCUSSION

Before data collection with the cans began, a series of tests were performed to see how accurate the prescribed method of obtaining an object's rotational inertia was in comparison to the known theoretical values. Objects used were a thin hoop, a thick hoop, and a solid cylinder. After multiple runs of each object, the experimental values were always larger than the theory by about 11%, which implies that the acceleration was less then it should have been. This could be due to viscous forces that were assumed negligible or other unknown sources.

Figure 3 is a plot of the calculated rotational inertia of the can down an incline as a function of the mass of water inside the can (where $g \sin\theta = 1.17$).

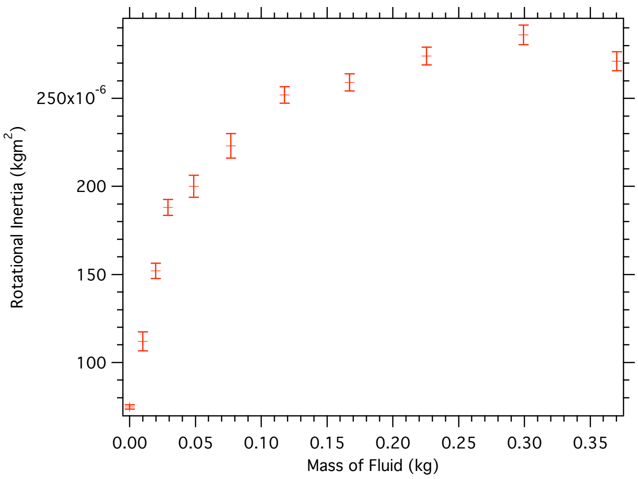

FIG. 3: Calculated rotational inertia as a function of water mass.

As the mass of fluid increases, the rotational inertia appears to approach a limit as the can is filled. No fit was applied to these data since no clear theory could be confidently applied.

Figure 4 is a plot of the acceleration of the center of mass down the incline as a function of the mass of water inside the can (using the same angle as before).

A dotted line is used to connect data points for visual aid and should not be misconstrued to be a fit to the data. Looking at the plot, a dip in acceleration can be seen for a small amount of fluid (about 10 mL). This corresponds to the increase in rolling time for a can containing 10 mL that was observed in figure 4 of K.A. Jackson's paper [2]. The following model (figure 5) is proposed in order to account for the increase in travel time down the incline.

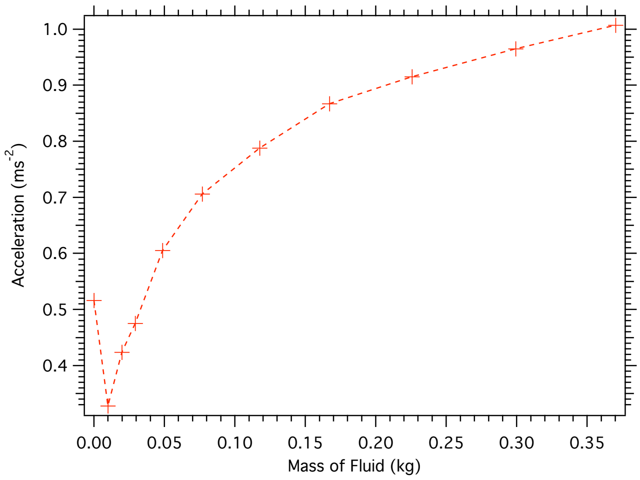

FIG. 4: Acceleration as a function of water mass.

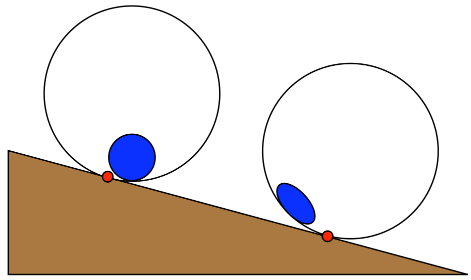

FIG. 5: Model of non-viscous and viscous fluids in a rolling can.

In figure 5, the object on the left has a non-viscous droplet inside of the can. As the can begins rolling without slipping down the incline, the droplet will slip without rolling and remain at the bottom of the can as shown. This could also model a scenario where the inside of the can is a strongly hydrophobic surface. This would mean that only a small amount of the water droplet actually comes in contact with the inner surface and thus is not as strongly affected by rotational movement of the surface as a less hydrophobic surface. Since the droplet will remain at the bottom of the can (downhill of the point where the can meets the incline's surface), a "forward" torque will be exerted by the droplet onto the surface of the can by exerting its gravitational force as a torque in the direction of the rolling motion of the can.

The right side of figure 5 represents a can with a viscous droplet, or water on a less hydrophobic surface than that of the can discussed earlier. In this scenario, the droplet is dragging along the surface of the can and is most likely rotating the fluid since it is adhering to the surface. As it sticks to the surface, it eventually reaches a position on the incline such that the angle is steep enough for the drop to roll. It is predicted that it will reach an equi-

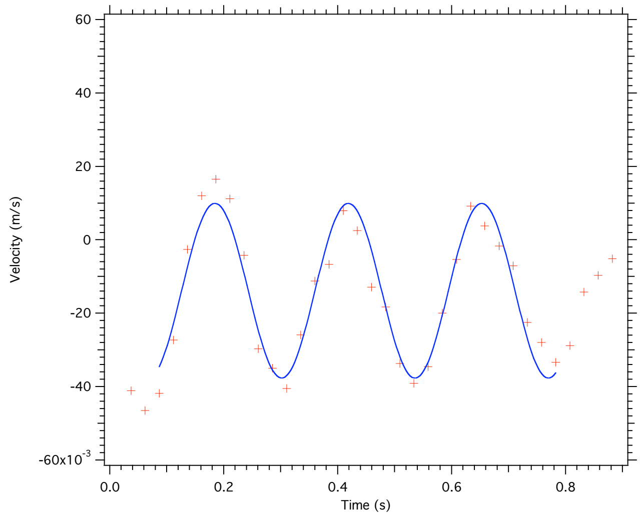

FIG. 6: Oscillation in the velocity of a can using 50 mL after subtracting the linear velocity versus time dependence.

TABLE I: Oscillatory motion of cans with fluid.

Fluid Amount (mL)	Frequency(Hz)
30	22.6 ± 0.2
50	26.6 ± 0.3
80	28.4 ± 0.3

librium point somewhere at a point uphill from where the can meets the surface of the incline. As the droplet remains in this position in the can (rolling without slipping), it is creating a "backwards" torque (acting in the direction opposite of the rolling can). This torque would therefore decrease the system's linear acceleration.

Oscillatory motion was clearly observed in the motion of the can with fluid when 30-80 mL of water were present. Once a linear fit ($y = a + bx$) had been performed on the velocity vs. time data, the line was subtracted from the data (with some modification of the slope to obtain horizontal sine waves), the frequencies of oscilation could be calculated using a sinusoidal fit ($y = y_0 + A \sin(fx + \phi)$). Figure 6 shows a sinusoidal fit to the first data collection using 50 mL. Table I lists the various amplitudes and frequencies of oscillation given the amount of fluid.

The oscillatory motion could be due to the fluid in the can adhering to the surface of the can until a large enough angle is reached. At this point, the amount of torque is large enough (since the mass of the water is greater than the mass of the empty can) to reverse the motion of the can and, in doing so, allows the fluid in the can to rush forward. Once the fluid is in the front of the can, the torque then allows the can to roll forward once again at an increased velocity. The body of the can again rolls ahead of the fluid and this process is repeated. This can be visually represented by the fluid "sloshing" back and forth in the can at some frequency.

The frequencies for each amount of fluid are well defined for each fluid. The amplitudes seem to have more variation, but this could be due to different starting conditions (some starting slightly more abruptly than others possibly).

CONCLUSIONS

When fluid is present in a can, its rotational inertia cannot be predicted using rigid body approximations. As more fluid is added, the rotational inertia of the system appears to approach a limit.

A decrease in acceleration is observed when the cans have approximately 10 ml of water. This is theorized to have been due to a torque opposing the motion of the can.

Oscillatory motion in the velocity can be observed when 30-80 mL of water are present. This is theorized to be due to a back and forth motion of the fluid in the can.

[1] David Halliday, Robert Resnick, Jearl Walker, *Fundamentals of Physics* (New Jersey, 2001), 6th ed., p. 223-231, 347-356.

[2] K.A. Jackson *et al.*, Amer. J. of Phys. **64**, p.277-282 (1996).