Measuring the surface tension of water from the diffraction pattern of surface ripples

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The surface tension of water was measured by setting up surface ripples with an oscillating wire attached to a speaker driven by a function generator. Exploiting the fact that the ripples behave like a diffraction grating, a laser was reflected off the surface and the interference spots measured from a digital image. This technique permitted accurate measurement of the spots, and eliminated most of the fluctuations due to vibrations which were not completely isolated by the air table. The power relationship between q and ω was verified, and the surface tension was measured to be $\sigma = 75.4 \pm 0.9\,\mathrm{mN/m}$, a 3.7% error from the accepted value of $72.75\,\mathrm{mN/m}$, and within 3σ [1].

I. INTRODUCTION

By setting up ripples on the surface of water, the surface tension can be measured. A small wire connected to a speaker cone driven by a function generator sets up the surface ripples, and a HeNe laser is angled at the water surface. The ripples act as a diffraction grating, yielding interference spots on a screen. By varying the frequency and amplitude of the function generator, the variation in the interference spot spacing can be measured, and from this, the surface tension can be obtained.

Surface tension arises from the fact that an imbalance of forces at the interface of a liquid causes it to behave like a stretched membrane. The tension in this membrane is known as surface tension [2]. The combination of gravity and surface tension allows a liquid to support the propagation of surface waves. In addition to surface waves, the bulk volume of a liquid also allows elastic waves [3]. This experiment focuses only on surface waves.

In 1894, Lord Rayleigh measured the surface tension of water by analyzing ripples [3]. Rayleigh's result of $74\,\mathrm{mN/m}$ was remarkably close to accepted value, $72.75\,\mathrm{mN/m}$, of the surface tension for a water-air interface at $20\,^\circ\mathrm{C}$ [1].

II. THEORY

Klemens [4] derived the dispersion relation for waves on liquid surfaces,

$$\omega^2 = \left(gq + \frac{\sigma}{\rho}q^3\right)\tanh(qD), \qquad (1)$$

where ω is the angular frequency, g is the gravitational acceleration, q is the wavenumber, σ is the surface tension, ρ is the mass density of the liquid, and D is the depth of the liquid. The gq term is due to gravity, while the $(\sigma/\rho)q^3$ term is due to surface tension. The $\tanh(qD)$ factor is approximately 1 for a deep reservoir and/or large wavenumber.

Klipstein [5] showed that surface ripples behave like a diffraction grating, and that the effective grating spacing

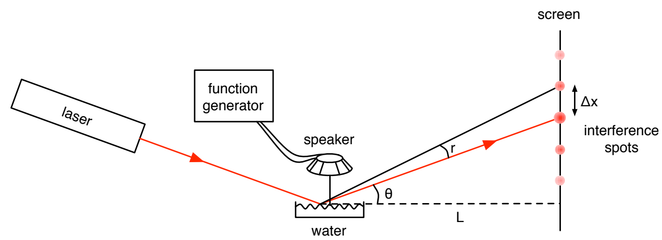

FIG. 1: A laser beam is aimed at a reservoir containing water, with ripples induced by a wire attached to a speaker driven by a function generator. The surface ripples act as a diffraction grating causing the interference spots on the screen.

λ_s of a surface wave is given by

$$\lambda_s = \frac{2\pi}{q}\sin\theta,$$

where θ is the angle of the laser beam measured from the horizontal axis (Fig. 1). This causes the light to be diffracted by an angle r satisfying [5]

$$\lambda = \frac{2\pi}{q}\sin r \sin\theta,$$

where λ is the laser wavelength. Using the small angle approximation, we obtain

$$q = \frac{2\pi}{\lambda} r\theta, \qquad (2)$$

where r and θ are in radians. Weisbuch [6] states the relation

$$q = \frac{2\pi}{\lambda}\sin\left(\frac{r}{2}\right)\left[\sin\left(\theta - \frac{r}{2}\right) + \sin\left(\theta + \frac{r}{2}\right)\right],$$

which also reduces to Eq. 2 using the small angle approximation. From Fig. 1, we see that

$$r \approx \tan^{-1}\left(\frac{\Delta x}{L}\right),$$

where Δx is the distance between adjacent interference spots.

In this experiment, λ is of order 10^{-7} m, r is of order 10^{-3}, θ is of order 10^{-2}, and D is of order 10^{-2} m. Thus

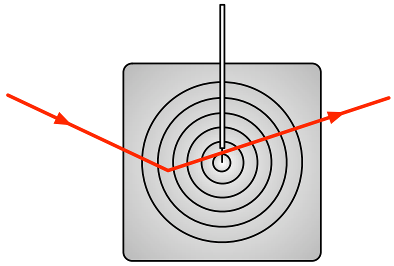

FIG. 2: The optimal spot to aim the laser is slightly off center and away from the wire.

qD is of order 10^1 and so the $\tanh(qD)$ term in Eq. 1 is simply 1 and we have

$$\omega^2 = gq + \frac{\sigma}{\rho} q^3.$$

Similarly, we note that the gq term due to gravity is of order 10^4 while the surface tension term $(\sigma/\rho)q^3$ is of order 10^{10} to 10^{13}. Thus we can neglect the gravity term and simply use

$$\omega^2 = \frac{\sigma}{\rho} q^3. \tag{3}$$

III. PROCEDURE

Before any measurements were taken, the air table was verified to be level. The θ and L measurements are independent of the ripples, so they were measured first. To measure θ, the height of the reflected beam was measured for various distances away from the water. Then θ can be obtained as the inverse tangent of the slope of a linear fit to the height versus distance plot. The distance L was obtained by leveling the He-Ne laser, aiming it directly over the reservoir, and then with a taut measuring tape, measuring the distance from the laser spot on the wall to the laser beam reflection point on the water surface. The speaker was connected to the Pasco PI-9587C function generator, and a small wire was connected to the speaker cone so as to extend into the water. As the speaker was driven by the sine wave output of the frequency generator, the wire moved up and down on the surface of the water. Due to the surface tension, this sinusoidal motion of the wire sets up ripples on the water surface.

Next, the laser was angled at the water, such that it hit a region shown in Fig. 2. The laser must hit slightly off center so as to miss the wire, but since the ripples must be perpendicular and not circular, the laser must be aimed some distance away from the wire. Furthermore, the capillary rise at the contact point must be avoided. However, if the laser is aimed too close to the wall of the reservoir, then ripple reflections could interfere and add extra noise to the interference spots. The optimal laser angle was found by varying the frequency of the function generator and adjusting the laser so as to optimize the clarity and frequency range of the interference spots. If the angle was too small or too large, the frequency range was found to decrease.

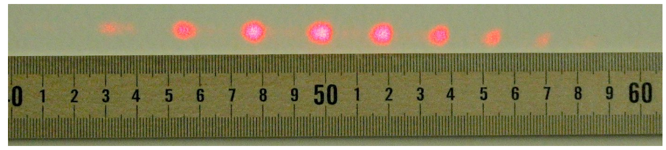

FIG. 3: A typical photograph of the interference spots.

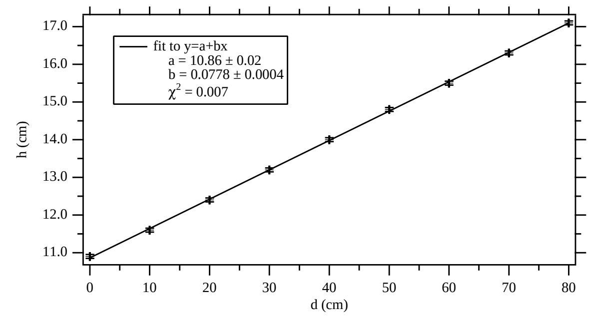

FIG. 4: Laser height h versus distance d fit to a line to determine θ.

A variety of reservoirs were tested for stability of the interference spots on the screen. It was found that for a small thin square reservoir, the laser beam was more stable on the screen than for a large deep circular reservoir. However, some fluctuation due to vibrations was still noted, even with the use of an air table. In order to accurately measure the distance between interference spots, a digital camera was used to photograph the spots. The images were loaded into Igor Pro for analysis. Figure 3 shows a typical image of the spots.

In order to measure Δx from the image, first the correct scaling factor was calculated. This was done by placing the cursors in Igor onto two ruler marks separated by a distance of 10 cm. The distance between the cursors in pixels was calculated, giving the conversion factor between pixels and meters. To measure Δx, the cursors were then placed on the centers of two spots the same order away from the reflection spot. The distance between these dots was calculated and divided by the number of spot spacings between the measured dots. To obtain an uncertainty estimate, Δx was measured between consecutive dots, and the standard deviation calculated and divided by the number of spot spacings to give $\delta(\Delta x)$.

The frequency of the function generator was varied between 50–850 Hz, with the amplitude adjusted to optimize the interference pattern. At each frequency, a photograph was taken for computer analysis.

IV. ANALYSIS & RESULTS

Using the method described above to measure θ, we obtain Fig. 4. A weighted fit to the line $y = a + bx$ was

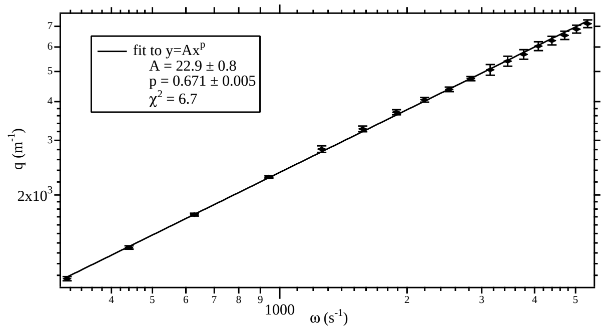

FIG. 5: The power rule relation between q and ω shown on a log-log scale.

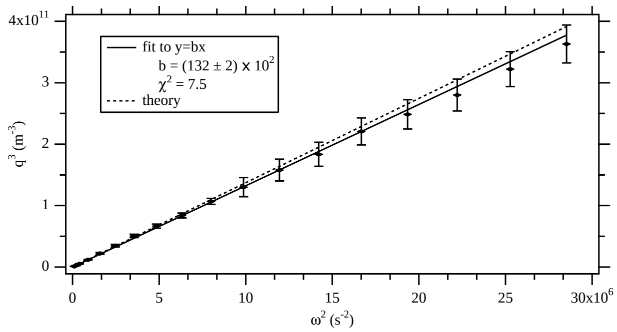

FIG. 6: Plot of q^3 versus ω^2 fit to a line (solid) along with a line (dashed) corresponding to the expected slope of ρ/σ.

applied to the data so that $\theta = \tan^{-1} b$, and thus the uncertainty in θ is given by $\delta\theta = \delta b/(1+b^2)$. This yields $\theta = 0.0777 \pm 0.0004\,\text{rad}$.

In order to verify that Eq. 3 holds, we plot q versus ω on a log-log plot (Fig. 5). Rearranging Eq. 3 we have

$$q = \left(\frac{\rho}{\sigma}\right)^{1/3} \omega^{2/3}.$$

Applying a weighted fit to the power rule $y = Ax^p$ while allowing A and p to vary, we obtain $p = 0.671 \pm 0.005$ as expected, and thus verifying the relationship between q and ω.

In addition to verifying the power rule relation, we can calculate the surface tension by rearranging Eq. 3 so that

$$\frac{q^3}{\omega^2} = \frac{\rho}{\sigma}.$$

Figure 6 shows a plot of q^3 versus ω^2 with a weighted fit to the line $y = bx$. Thus $\sigma = \rho/b$, and so $\delta\sigma = (\rho/b^2)\delta b$. This yields $\sigma = 75.4 \pm 0.9\,\text{mN/m}$.

V. CONCLUSION

The surface tension of water was measured by setting up surface ripples with an oscillating wire connected to a speaker driven by a function generator. Exploiting the fact that the ripples act as a diffraction grating, a laser was reflected off the surface and the interference spots measured from a digital image. This technique allowed accurate measurement of the spots and eliminated most of the fluctuations due to vibrations not completely isolated by the air table. While keeping the laser angle constant, the frequency of the function generator was varied between 50–850 Hz. The power law relationship between q and ω was verified; a fit to the data yielded $q = A\omega^{0.671\pm0.005}$ where we would expect the power to be 2/3. Furthermore, with the power fixed at 2/3, the surface tension was measured to be $\sigma = 75.4 \pm 0.9\,\text{mN/m}$, a 3.7% error from the accepted value of 72.75 mN/m, and within 3σ.

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