The Spectrum Analyzer and The Mode Structure of a Laser

Sahibzada Amir Hassan[a)] Advisor: D. T. Jacobs[b)]
Physics Department, The College of Wooster, Wooster, Ohio 44691

(Written April 31 1997)

Theis experiment reveals that the output of a laser is composed of range of frequencies, which is, theoretically contained within a Gaussian distribution. The difference in frequency of neighbouring frequencies in this range is Δv, called the mode separation; a constant given by the equation $\Delta v = \left(\frac{c}{2L}\right)$, where c is the speed of light and L is the length of the laser cavity. The mode separation of a Melles Griot He-Ne laser, with a principal (main) emission wavelength of 543.5 nm, was experimentally determined (using a spectrum analyzer in conjunction with a Fabry-Perot Interferometer) to be approximately $0.377 \pm 0.013\,\mathrm{GHz}$. Using the expression for the mode separation, the length, L, of the laser cavity was calculated to be approximately $0.398 \pm 0.01\,m$. Which is in error of about 1% from the value of L calculated for the published[6] value of the mode separation. © *1997 The College of Wooster*

INTRODUCTION

The acronym laser means **l**ight ***a**mplification* by ***st**imulated* ***e**mission* of **r**adiation. The words 'amplification' and 'stimulated emission' refer to the process of the interaction of electromagnetic radiation with matter, proposed by Einstein.[2]

Laser light[1] is not an ideal *monochromatic* light source; that is, the output of a laser is not a an electromagnetic radiation of a single frequency. Instead, as the experiment will reveal, laser light consists of discrete frequency components. We can, however, determine how close the laser light may be to a single frequency using a Fabry-Perot Interferometer.[1]

The Fabry-Perot Interferometer consists of a pair of mirrors, with inner surfaces highly reflective to laser light, whose separation (or the *optical cavity* [1]) can be varied. As a beam of collimated (coherent) light, such as a laser, enters the interferometer it bounces back and forth between these plates. If the original wave and the reflected wave are *in phase* then they reinforce each other and *optical resonance* occurs. Waves that are in phase replicate themselves and their electric fields add such that the energy density of the resulting wave is high enough to allow transmission through the reflecting mirrors (Fig. 1).

The condition for a self replicating field (optical resonance) is that the distance between the two mirrors is equal to an integral number of half wavelengths. Therefore, for any specific separation of the two mirrors there may exist a number of self replicating fields, or *longitudinal modes*. Conversely, only for certain frequencies is the cavity *resonant.* For these longitudinal modes, or resonant frequencies, light is transmitted through the interferometer and detected using a photo-diode. Examining the profile of the output would reveal the "spectral content"[1] of the incident laser.

This experiment will present an analysis of the structure of the laser output from a helium-neon (He-Ne) Laser device.

Theory

The laser resonator[2] (cavity) follows the same basic principle as the Fabry-Perot interferometer cavity. However, the laser resonator differs in functionality due to the presence of an amplifying medium[1] between the mirrors and the fact that the length of the laser cavity (or *resonator)* L cannot be varied.[1] More details may be found in the referenced texts or in any standard optics textbook.

When the incident and reflected light waves are 'self replicating' or *in phase*[1] within the laser cavity the electric fields of both the waves add up due to *onstructive' interference*[1] . The amplitude and consequently the energy density of the resulting wave increases such that almost 100% of the resultant wave is transmitted through the mirrors. For the laser cavity to be resonant the following relationship must be satisfied;

$$L = m\left(\frac{\lambda}{2}\right) \quad (1)$$

That is the distance L must be an integral multiple of half wavelengths. Since m can be any integer there may exist a number of wavelengths (or frequencies) which might satisfy the relationship. From equation (1) it follows that each field for which the laser cavity is resonant, for a fixed separation L, may be expressed in terms of the frequency of light,

$$v_m = \frac{c}{\lambda_m} = \frac{c}{2L/m} = m\left(\frac{c}{2L}\right) \quad (2)$$

Where c is the speed of light and m is any integer. The separation between neighboring transmitted frequencies, the *mode spacing*, is given by,[2]

$$\Delta v = v_{m+1} - v_m = (m+1)\frac{c}{2L} - \frac{mc}{2L} = \frac{c}{2L} \quad (3)$$

As the laser resonator cavity and the Fabry-Perot cavity are identical, by applying the same kind of formalism, we can show that the expression for the difference between neighboring resonant frequencies set up in the Fabry-Perot cavity is,

$$\Delta\sigma = v_{m+1} - v_m = (m+1)\frac{c}{2d} - \frac{mc}{2d} = \frac{c}{2d} \quad (4)$$

Where d is the separation between the mirrors in the Fabry-Perot Cavity. Since d can be varied, the Fabry-Perot interferometer can accommodate different integral half wavelengths at different values of the separation d.

When the separation d is increased considerably such that $\Delta\sigma$ is large enough, only one frequency would be allowed to be transmitted and observed within the range of observable frequencies of the interferometer thereby allowing us to analyze the frequencies contained in the incident laser light. [2]

If the difference between the neighboring transmitted frequencies (longitudinal modes) is a constant then Δv (eq. 5) is equivalent to the mode spacing of the laser input.

It can also be shown[2] that changes in d greater than $\frac{\lambda}{2}$ would then result in the same frequencies being transmitted as observed between Δd = 0 and $\Delta d = \frac{\lambda}{2}$. Thus, the range of frequencies observed, as the mirror separation d is varied, without the pattern repeating itself is called the *free spectral range* (FSR) of the interferometer, given by,

$$\Delta\sigma = \frac{c}{2d} \quad (6)$$

According to Maxwell [2] there exists a 'distribution' of velocities of the atoms in a gaseous amplifying medium (He-Ne gas in our case) is given by a Gaussian (Maxwellian) [2] with the most probable velocity given by;

$$v_p = \left(\frac{2kT}{M}\right)^{1/2} \quad (7)$$

Where M is the atomic mass of the amplifying gas medium, k is the Boltzman constant[2] and T is the temperature at which the laser transition (*lasing*) occurs. This causes *Doppler broadening*[2] to occur within the laser resonator and consequently the output signal consists of a band of frequencies contained (theoretically) within a Gaussian profile rather than a single frequency (coherent) output. Therefore, the laser output can be characterized as an intensity distribution given by;[3]

$$I(\nu) = I_0 \; e^{-\left[\frac{c(\nu-\nu_o)}{\nu_0 v_p}\right]^2} \quad (8)$$

The full width at half maximum(FWHM)[3] of this distribution is given by equation (9)[4].

$$\Delta v_{HM} = \frac{\left(2\sqrt{\ln 2}\right)\nu_o v_p}{c} = \sqrt{(8\ln 2)\frac{kT}{Mc^2}} = 2.35\sqrt{\frac{kT}{Mc^2}}v \quad (9).$$

Therefore, after examining the spectral content of the laser, using the Fabry-Perot Interferometer, we can interpolate a Gaussian line fit (using IGOR) of the form $y = k[0] + k[1] \; e^{\left[\frac{x-k[2]}{k[3]}\right]^2}$ through the values of the resonant frequencies and their corresponding intensities to verify the validity of these assumptions.

If the expression given above is used for interpolating the data then the line-fit parameters can be compared to parameters in equation(8) to give the following correlation. k[0] should be equal to zero, since no intensity should be observed for frequencies lying outside the Gauusian distribution. The parameter k[1] should be equivalent to the maximum intensity I_o that may exist within the Gaussian profile. K[2] should correspond to the central frequency v_o of the line shape and k[3] should be numerically equivalent to $\nu_o v_p/c$, from which the FWHM can be calculated. Once the FWHM is known, for a given the line width and relative molecular mass of the gaseous gain medium, the lasing temperature T for the 543.5 nm He-Ne laser transition can be determined using equation (9). Further information of the theoretical model may be found in reference 4.

EXPERIMENT

The apparatus which consisted of a Melles Griot model 05 SGR 871 GreNe™ Helium-Neon laser,

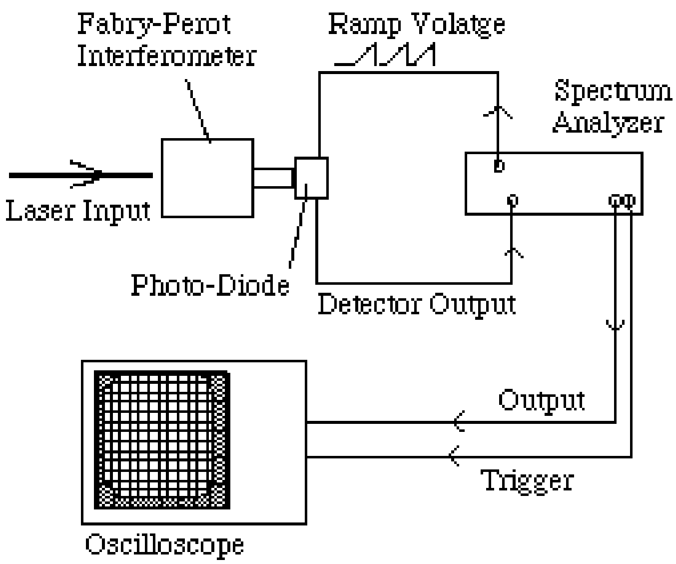

Figure 1: Schematic Diagram of the apparatus.

The intensity of the resonance peaks (in mVolts) and their respective frequencies can be 'read off' the Oscilloscope screen using the cursor option (Fig. 2). The two cursors, namely V1 and T1, were used as reference (fixed) axis and the values of the voltages of the peaks (modes) and their corresponding times are measured, using cursors V2 and T2 respectively (Fig. 2). This

The Intensities of the resonance peaks (in mV) and their respective times were collected and analyzed using the IGOR pro ® software.

ANALYSIS AND INTERPRETATION

The calibration of the oscilloscope revealed that 7.42 ms on the oscilloscope corresponded to 7.5 GHz (FSR). Thus 1 ms on the time divisions of the oscilloscope corresponded to approximately 1.06 ± 0.01 GHz of the actual signal input.

The pattern of the resonance peaks, observed on the oscilloscope, was continuously varying in size. The pattern of the resonance peaks was

a Coherent Spectrum Analyzer controller 251, a Coherent Fabry-Perot interferometer (Model 240-1-A) and a Hewlett Packard 54600 oscilloscope, was set up as shown in figure 1. The intensity of the transmitted frequencies from the Fabry-Perot Interferometer (measured in mVolts) were detected using a photo diode and displayed on the HP oscilloscope. Details of the experimental procedure may be found in ref. 4 and ref. 5.

method also allows us to measure the mode spacing of the resonance pattern directly. It is imperative that the FSR should be greater than the line width of the laser output, so that all the possible longitudinal modes are within the scanning range of the interferometer.

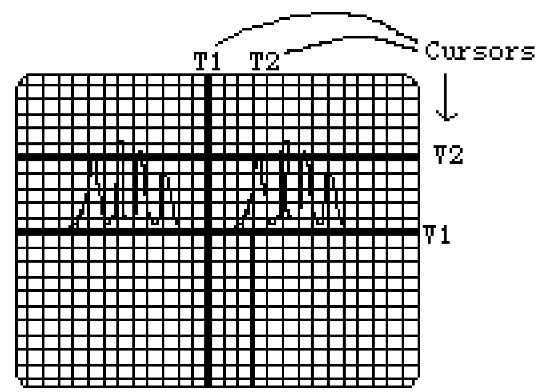

Fig 2: Identical patterns of resonant frequencies. On screen cursors are used to determine the intensities (measured in mV) and the corresponding times. V1 and T1 used as reference axis.

'skewed' in one direction and then the other, possibly, indicating thermal instability.[6] Since, during the warm up process the length of the laser resonator cavity expands the longer mode cavity may be the cause of the schematic shift in the observed resonance curves.[6] Therefore, the data was not collected until the pattern was symmetric. Thereafter, the voltage values and the time values were collected (as shown in Table 1). Since the time values for the successive resonance peaks are measured in ms, any measurement for the time must therefore be multiplied by the scaling factor to give the corresponding frequency of the signal input.

The time base of the oscilloscope was set to 200 μs per/div. for better resolution of the trace.

Therefore, the error involved in reading the voltage values off the oscilloscope screen was reduced.

n - Peak #	Δv _ms	Δv _GHz
2 & 3	0.356	0.377
3 & 4	0.356	0.377
4 & 5	0.356	0.377

Table 2: The mode spacing between neighboring resonance peaks in the transition line shape (scaled to GHz).

The values of the mode-spacings were found to be very consistent as shown in table 1.

Since the times corresponding to the resonance curve was approximated by visual inspection of the signal trace on the oscilloscope, the readings in table 2 may be omitting the human error that may have been present in the data set.

'ΔV (mv)'	'ΔT (msec)'
3.125	-0.008
50	0.348
126.6	0.704
128.1	1.06
48.44	1.416
3.125	1.68

Table 3: The values of the voltage of the resonance peaks and their corresponding times.

Plotting the data set given in table 3, the following graph was obtained on IGOR pro. The Gaussian curve fit of the form $y = K[0] + K[1]\ e^{\left[\frac{x-K[2]}{K[3]}\right]^2}$ was used to interpolate the data points and the following curve-fit was obtained. The curve fit shows that the resonance peaks are indeed contained within a Gaussian distribution. The parameter k[0] was not held at 0 (for the first fit) as predicted by the theory since the background noise prevented the intensity to fall to zero on either side of the resonance pattern as reflected in the first and the last data points.

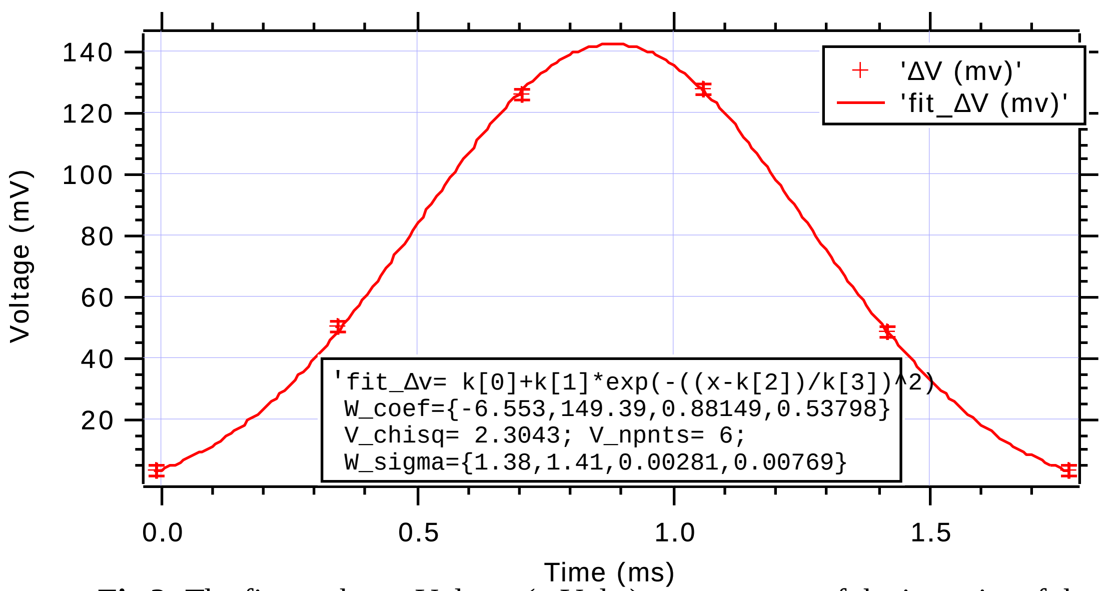

Fig 3: The figure shows Voltage (mVolts) as a measure of the intensity of the frequencies observed vs. their corresponding times (in ms). The curve used to interpolate the data points is a Gaussian of the form: (k[0] not held equal to zero).

$$I = K[0] + K[1]*e^{\left[\frac{x-K[2]}{K[3]}\right]^2}$$

The parameters of the second curve-fit (holding the k[0] constant at zero) agree considerably with the first curve-fit.

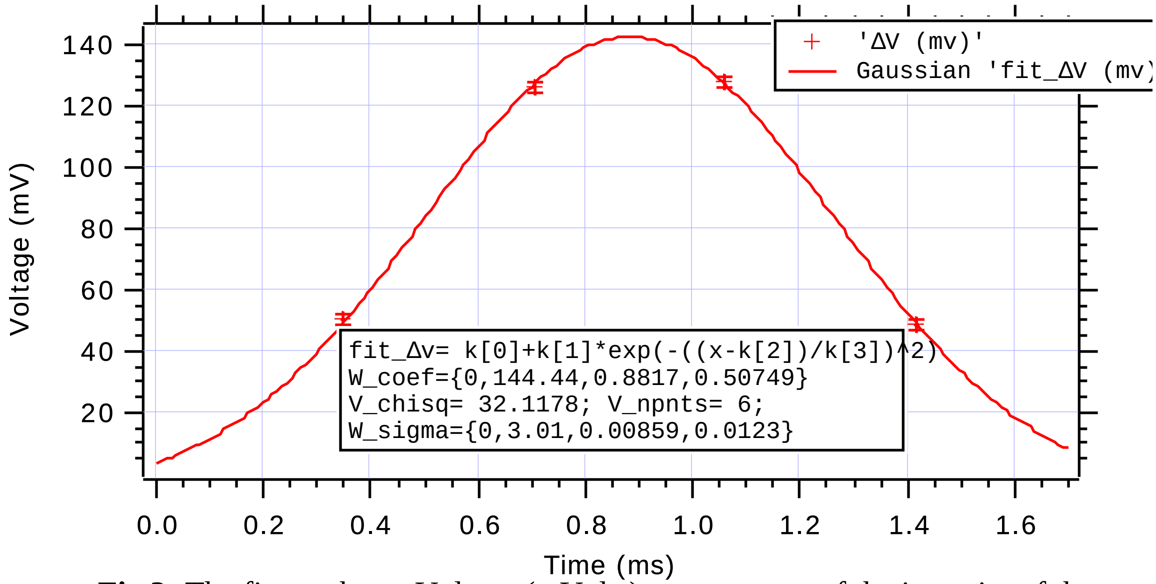

Fig 3: The figure shows Voltage (mVolts) as a measure of the intensity of the frequencies observed vs. their corresponding times (in ms). The curve used to interpolate the data points is a Gaussian of the form: (k[0] held equal to zero).

$$I = K[0] + K[1]*e^{\left[\frac{x-K[2]}{K[3]}\right]^2}$$

Using the correlation of the line fit to the Gaussian distribution for the gas medium we can calculate the following,

From curve fit I:

Parameters Values from Gaussian Curve-fit. k[0] determined freely.	Theory.
K[0] = -6.6 ± 1.4	(K[0] = -6.6 ± 1.4)
K[1] = 149.4 ± 1.4	I_o =149.4 ± 1.4
K[2] = 0.881 ± 0.003	v_o = k[2]*1.06= 0.933 ± 0.003 GHz
K[3] = 0.54 ± 0.01	$\nu_o v_p/c$= 0.54 ± 0.01 *1.06 = 0.57 ± 0.01 GHz

From curve fit II:

Parameters Values from Gaussian curve-fit. (holding k[0] =0.)	Theory
K[0] = 0	(K[0] = 0)
K[1] = 144.4 ± 3.01	I_o =144.4 ± 3.1
K[2] = 0.882 ± 0.001	v_o = k[2]*1.06= 0.934 ± 0.001 GHz
K[3] = 0.51 ± 0.01	$\nu_o v_p/c$= 0.51 ± 0.01 *1.06 = 0.54 ± 0.01 GHz

The value of chi squared for the second set of line fit parameters (for Fig. 10) is 2.3 as compared to 32.1 in the case when k[0] was fixed at zero. The value of the free parameter K[0] could be interpreted qualitatively as a vertical translation of the standard distribution curve. That is, the Gaussian line fit ought to have decreased exponentially towards the value of the baseline (observed on the oscilloscope) of approximately 3 mV on either side of the resonance pattern. However, the parameters from the second line fit would suggest that the respective Gaussian would tend to -6.6 $\pm$ 1.4 mV on either side of the data points. This discrepancy arises due to the fact that the first and the last data points in Table 1 are not resonance peaks. They were determined arbitrarily to provide enough data for a successful Gaussian line fit. If these data points on the base line were defined far enough from either side of the resonance pattern, the corresponding line fit would yield a k[0] parameter sufficiently close to the observed base line of 3 mV. Despite these differences the parameter values for k[2] or the principal frequency agree remarkably for both curve fits.

Given that the He-Ne laser utilizes a 543.5 nm laser transition[6], given that the atomic mass of neon is approximately 20 a.m.u, the lasing temperature T is calculated as follows;
(Note that the atomic mass of neon was only used since He metastable atoms only serve to transfer energy to the excited neon atoms, which in turn are responsible for the actual laser output).[7]

Since the line width value has been calculated from the line fit parameters, therefore, we can determine the value for the lasing temperature for the 543.5 nm transition as follows.
Using equation (9)

$$\Delta v_{HM} = \frac{\left(2\sqrt{\ln 2}\right)\nu_o v_p}{c} = \sqrt{(8\ln 2)\frac{kT}{Mc^2}} = 2.35\sqrt{\frac{kT}{Mc^2}}\,v_o$$

$$\left(\frac{\left(2\sqrt{\ln 2}\right)\times k[3]}{2.35}\right) = \left(\frac{v_o}{c}\right)\times\sqrt{\frac{kT}{M}}$$

$$\left(\frac{\left(2\sqrt{\ln 2}\right)\times(0.57\pm 0.01)\times 10^9}{2.35}\right) = \lambda\times\sqrt{\frac{kT}{M}}$$

$$T = \left(0.709\times(0.57\pm 0.01)\times 10^9\times 5.4\times 10^{-9}\right)^2\frac{M}{k}$$

$$T = 118.86\pm 0.07\ \mathrm{K}$$

By inspection of equation (8) we can see that the line width for the Gaussian line shape is directly proportional to the resonant frequency v_o. That is, a decrease in the lasing wavelength (632.8 nm to 543.5 nm) should result in an increase in the corresponding frequency . That would imply that the line width of the Gaussian for the 543.5 nm transition would be relatively larger. From the line fit parameters we can see that the calculated line width is smaller; 0.6 GHz as compared to 1.5 GHz for the 632.8 nm transition.[5] Since the line width value is incorrect, therefore, the value for the lasing temperature T is incorrect.

Furthermore 118.86 K is below room temperature and it was deduced from observation that the laser output occurred after the laser had been warmed up for a couple of minutes at room temperature which further invalidates the calculated value of T.

By contrast, direct measurements of the mode spacing, using voltage and time cursors, revealed that the mode spacing (calculated using the data in Table 4) was in fact (0.356$\pm$ .002)ms * 1.06 (GHz / ms) = 0.377$\pm$0.002 GHz or approximately 377 MHz. The published[6] value is given to be approximately 380 MHz. Thus the measured value is in error of about 0.8% from the published[6] value.
Furthermore, as the relationship between the mode separation and the cavity length is given by equation (3) measured value of Δv can be verified as follows.

Since $\Delta v = \left(\frac{c}{2L}\right)$, substituting the values of c, the speed of light (3x10[8]), and the mode separation Δv an approximate value for the cavity length can be calculated.
That is,
$L \approx \frac{c}{2\Delta v} = \frac{3\ 10^8}{2\ (377)} = 0.398 \pm 0.01m$ which is in error of approximately 1% from the value of the actual cavity length (determined by a similar calculation for the published[6] Δv value).

CONCLUSION

The experiment was conclusive in determining and verifying existence of longitudinal modes in the output of a laser. By measuring experimentally the separation of the spectral frequencies contained in the laser emission, we were able to verify that these frequencies differ by a constant of magnitude $\Delta v = \left(\frac{c}{2L}\right)$; the mode separation of the laser output. Moreover, since the value of c, the speed of light, and the value of the L; length of the optical cavity are known, the above relationship enables us to verify the value of the mode separation. In fact the calculated value of the length L differed from the actual[6] value by only 0.8%.

On the other hand the spectral frequencies observed were not entirely contained within a Gaussian distribution (as seen in the data analysis). The resonance pattern obtained through the Fabry-Perot interferometer was seen to be skewed or shifted at different intervals of time (as shown in Fig. 12). This effect may be attributed to the thermal instability [6] of the laser cavity. The calculation for the lasing temperature may have been flawed because of two possible reasons. Firstly the Doppler line width is greater than the natural line width of the profile. Secondly, the transition line shape is more accurately modeled by the Voigt line profile

$$I(\nu) = (\text{Const})\int_0^\infty \frac{e^{\left\{-\left[\frac{c(\nu'-\nu)}{\nu_o v_p}\right]^2\right\}}}{(\nu-\nu')^2+(\Delta\nu/2)^2}\,d\nu' \qquad (10)$$

Furthermore, one might improve the experiment by verifying whether the laser is in fact operating in the TEM_{OO} mode by inserting a diverging lens between the laser and the interferometer and observing the effect on a cardboard placed behind the lens. If the laser is in fact operating in its TEM_{OO} mode then diverging lens should cause the laser output to appear as shown in figure 11. A "uniform, rotationally symmetric, spot devoid of any internal nodes or lines signifies that the laser is" in fact operating in its TEM_{OO} mode."[1]

Further study might include replacing the 0.67 mW He-Ne laser with a more powerful laser, which is more thermally stable, such that more longitudinal modes can be observed and consequently the Gaussian shape of the spectral resonance frequencies can be determined with more accuracy. If the Gaussian behavior of the laser transition line shape can be established then a better estimate of the value of the lasing temperature T can be calculated using the equation (9).

ACKNOWLEDGMENTS

1 O'shea, Donald C., Callen, W. Russell and Rhodes, T. Rhodes in Introduction to Lasers and their Applications, (Addison Wesley, Reading, 1978). Pg. 33-92.

2 Pedrotti Frank L and Pedrotti Leno S. in, Introduction to Optics, 2nd ed. (Prentice Hall, New Jersey, 1993), Pg. 426-440.

3 McGrawHill Encyclopedia of Physics, Sybil, Sybil P editor in chief.- 2nd ed. (McGraw-Hill, New York, 1993), Pg. 690.

4 Garg, Sheila, Modern Optics Project #3 Assignment Literature, The College of Wooster, OH, 1997 (unpublished).

5 Physics 401, Junior Independent Study Manual, Spring 1997. Pg. 30.

6 Phillips, Richard A., and Robert D. Gherz, "Laser Mode Structure Experiments For Undergraduate Students," Am. J. Phys. **38** 429-432 (1970).

7 Melles Griot Optics Guide 4 (product catalogue), copyright 1988. Pg. 17/28 - 17/29.

8 Handbook of Optics Volume I, Michael Bass editor in chief.- 2nd ed. (McGraw-Hill, New York, 1995), Pg. 11.32.