Moment of inertia and nutation of a captured gyroscope

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A captured gyroscope was used to measure the time period of the inner and the outer gimbals in oscillatory motion. From these measured values, the moments of inertia about the inner and the outer gimbal axes were calculated. These moments of inertia were used to calculate the nutational frequency of the gyroscope. Finally the free nutational frequency of the gyro was measured and then compared to that calculated . The moment of inertia for the outer gimbal was measured to be $I_{og} = (244.3 \pm 4.4)$ gm.cm^2. The moment of inertia about the inner gimbal axis was measured to be $I_{ig} = (169.8 \pm 5.6)$ gm.cm^2. The measured value for angular frequency of nutation $= (17.6 \pm 0.4)\, rad/\sec$. The calculated value for this angular frequency of nutation was (16.7 ± 1.3) rad/sec.

INTRODUCTION

The Gyro or gyroscope, contrary to public opinion, is an old instrument. It was invented by a French physicist by the name of Focault over one hundred years ago. Since that time, several items have been designed and developed to demonstrate that gyroscopic phenomena does exist and can be useful. Some one hundred years ago, using a pendulum, Focault developed a method of proving the earth's rotation. From Focault's beginning, many papers were written and distributed by other physicists which demonstrated how forces obtained from rotating bodies or wheels could be used to produce a stable axis or stable line of reference that was capable of resisting known forces, such as wind, gravity, earth's rotation and the like.

A rotational quantity associated with an axis, such as an angular velocity, angular momentum, or torque, can be represented by a vector along the axis. Thus angular velocity can be defined as a vector quantity ω with magnitude equal to the number of radians through which the body turns in unit time and direction along the axis of rotation. A vector can have either of two opposite directions along a given line (corresponding to an arrowhead on either end of the arrow representing the vector); it is customary to define angular velocity as having the direction in which a right-hand thread screw would advance if turned with the body. Alternately when the fingers of the right hand are wrapped around the axis with the fingers pointing in the axis of rotation, the thumb points in the direction of the vector angular velocity.

Objectives

1) Measurement of system moments of inertia: To experimentally measure the composite inner gimbal plus rotor moment of inertia about the inner gimbal axis of freedom and to measure the composite rotor, inner gimbal and outer gimbal moment of inertia about the vertical axis. For the latter measurement, the position of the inner gimbal would affect the measured value since the gimbal is not symmetric about its own axis of freedom. Therefore, the measurement shall be defined as taken with the rotor spin axis in a horizontal plane.

2) Analytical study of the phenomenon of nutation:[1] To verify Equation (1).

$$\omega_n = \sqrt{\frac{H^2}{I_{og} I_{ig}}} = \text{nutation frequency} \qquad ..(1)$$

Where H = Angular momentum, I_{og} = Moment of inertia of the outer gimbal, I_{ig} = Moment of inertia of the inner gimbal

EXPERIMENT

Equipment

Some modifications had to be made to the MITAC to generate the spring restraint to measure the inner and outer Gimbal moments of inertia. To measure the outer gimbal moment of inertia, the two springs were mounted across from each other. This provided the spring restraint for our Gyroscope. To measure the inner Gimbal moment of inertia, the springs were set up to allow the Inner Gimbal to move up and down about its horizontal axis. The springs have a spring constant of k which was measured using the Jolly balance and was determined to be 100 gm/cm.

Procedure

Outer Gimbal Moment of Inertia measurements

To measure the outer gimbal moment of inertia, the wheel or the rotor of the gyroscope was allowed to speed up until a constant angular speed was reached. At this speed, the Outer Gimbal of the gyroscope was nudged a few degrees and due to the spring restraints, the wheel kept going through its own oscillations about the horizontal axis. The time period for twenty of these oscillations was recorded and five such measurements were made. Further calculations to determine the moment of inertia were performed and these have been discussed in the calculations section of this report.

Equation (2) below has been used to calculate the moment of inertia of the Outer gimbals.

$$\sqrt{\frac{K}{I}} = \omega_0 \qquad ...(2)[1]$$

Where ω_0 is the angular frequency of the Outer gimbals, I is the moment of inertia of the rotor system about the outer gimbal axis. K is the combined torque constant of the system, where

$$K = 2ka^2 \qquad ...(3)$$

Where k is the spring constant of the two springs and a is the length of the lever arms (the width of the gimbals in this case).[1]

Experiment number	Time for 5 oscillations about Outer gimbal axis (seconds)	Time for 10 oscillations about Inner gimbal axis (seconds)
1	3.125	4.5
2	3.447	5.5
3	3.583	5.8
4	3.245	5.7
5	3.248	5.65
6	3.157	5.6

Table 1:Data for the time of oscillations used to find the moment of inertia about the inner and the outer gimbal axis

Inner Gimbal Moment of Inertia measurements

To measure the inner gimbal moment of inertia, the rotor was allowed to spin up as above and then the inner gimbal was given a push which made the gimbal oscillate about its vertical axis. As in the experiment above, the time period for 10 oscillation was recorded and further calculations were performed to obtain the moment of inertia. These have been explicitly performed in the sample calculations section.

Equation (2) also holds for the Inner Gimbal moment of inertia calculations, where ω_0 is the angular frequency of the Inner gimbals, K is the combined torque constant of the springs, I_{ig} is the moment of inertia of the rotor system about the inner gimbal axis.

Measurement of nutation of the gyro

Nutation, as described above, is also a very important property of a gyroscope. This can be calculated for our gyro from the angular momentum of the gyro and from the derived moment of inertia of the inner and outer gimbals. However, to verify our measurements and to help us understand the applications of the gyroscope, we have made separate measurements for nutation as well. For measurement of nutation frequency, we let the gyroscope rotor rotate freely without any elastic or viscous fluid obstructions. This allows the gimbals to rotate and revolve in a three dimensional fashion. However, due to the moment of inertia of the gimbals and the angular momentum of the rotating wheel of the gyro, there is a restoring force acting on the rotor and this forces the gyro to go through oscillations when it is disturbed from its equilibrium position. These oscillations are timed and the time period for twenty such oscillations was measured. This experiment was performed five times.

Experiment number	Time (seconds)	Number of oscillations
1	7.173	20
2	6.871	20
3	7.210	20
4	7.187	50
5	7.241	20

Table 2: Data for the period of oscillations used to find the nutation of a free gyroscope

ANALYSIS AND INTERPRETATION

Example calculations

Average time period for five oscillations in Data Table 1 for *outer gimbal* moment of inertia, $\bar{x} = 3.30 \pm 0.18$ seconds, where the error is one standard deviation. Where $\omega_0 = \frac{5}{\bar{x}} \times 2\pi$ =9.5 rad/sec. Using equation (3), K = torque constant = $2 \times (10.5\text{ cm})^2 \times 100$ gm/cm=22000 gm.cm. So, by equation (2), $I_{og} = \frac{22000}{9.5^2} = 244.3\,gm.cm^2$. After propagating the errors, the moment of inertia about the Outer gimbal moment of inertia = $I_{og} = (244.3 \pm 4.4)\,gm.cm^2$

Average time period of 10 oscillations in the setup for measuring *inner gimbal* moment of inertia, $\bar{x} = 5.65 \pm 0.44$ seconds, where the error is one standard deviation. Where $\omega_0 = \frac{10}{\bar{x}} \times 2\pi$ = 11.1 rad/sec. Using equation (3), K = torque constant = $2 \times (10.5\ \text{cm})^2 \times 100$=22000 gm.cm. So, by equation (2), $I_{ig} = \frac{22000}{(11.1)^2} = 169.8\, gm.cm^2$, After propagating the errors, the moment of inertia about the outer gimbal moment of inertia = $I_{ig} = (169.8 \pm 5.6)\, gm.cm^2$.

For calculation of nutational frequency of the gyro configuration, the calculations have been performed twice. In the first case, the information from the inner and outer gimbals has been used. The nutation frequency is also measured and this value is compared to this calculated value. Using equation (1) and the measured values for inner and outer gimbal moments of inertia, Nutational frequency (calculated) = (16.7 ± 1.3) rad/sec. From Data Table 3, the average value for the time period of twenty oscillations in the nutation cycle $\bar{x} = 7.14 \pm 0.36$ for 20 oscillations

Thus, using this measured time period, frequency of the nutation = $\frac{20 oscillations}{7.14 \sec} \times \frac{2\pi rad}{oscillation}$ = 17.6 rad/sec**.** Measured value for frequency of nutation $= (17.6 \pm 0.4)\, rad/\sec$, where the error is one standard deviation.

These calculated and measured results agree and confirm the theory for the gyroscope.[1] Comparing the results of the measured value for the frequency of nutation to the calculated value of the frequency from the inner and the outer gimbal moments of inertia, we find that there is negligible difference between the two values compared to our calculated errors.

CONCLUSION

In this experiment, a standard captured gyroscope was used to measure the inner and outer gimbal moments of inertia of this particular gyroscope. These two values were used to finally determine the nutational frequency of this gyroscope. However, to confirm our calculations and our measured values, the nutational frequency was measured. The measured value for nutational frequency $= (17.6 \pm 0.4)\, rad/\sec$. The calculated value for this frequency was (16.7 ± 1.3) rad/sec. The moment of inertia for the outer gimbal was determined to be . The moment of inertia about the inner gimbal axis was determined to be $I_{og} = (244.3 \pm 4.4)\ \text{gm.cm}^2$.

[1] *MITAC Handbook "The Classroom Gyroscope"* AC Spark Plug Division, General Motors Corporation, Michigan, 1959 This was a handbook and a general guide to experiments that could be performed by the MITAC, a versatile classroom gyroscope..

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