Modernizing the Cavendish Experiment

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Using a torsion balance one can calculate the gravitational constant *G* found in Newton's law of universal gravitation. This law states that the attraction between two objects is inversely proportional to the square of the distance between them. The apparatus consists of a horizontal pendulum, which hangs from a torsion wire and has two small spheres attached at either end. By bringing two larger spheres into the proximity of the two small spheres, the force of attraction can be observed. To quantify the observations a laser is reflected from a small mirror mounted on the pendulum. As the pendulum oscillates due to the attracting force, the angle about which it oscillates is determined by observing the position of the reflected laser beam. Using a position sensitive detector to act as a virtual ruler, the position of the reflected light beam is automatically detected as it oscillates, simplifying the data collection procedure. With this improved technique, I measured the gravitational constant to be $6.3 \pm 0.3 \times 10^{-11}\ \mathrm{Nm^2/kg^2}$ which only has a 5.1% difference from the accepted value.

Our world is ruled by two sets of laws: the laws of gravity and the laws of quantum mechanics.

-Prabhakar Gondhalekar, in *The Grip of Gravity*

INTRODUCTION

In 1798, Henry Cavendish performed one of the greatest experiments in the history of physics when he measured the gravitational constant. The problem arose when Newton developed his theory on the law of universal gravitation but was unable to calculate a value for *G*. With the use of a torsion balance Cavendish measured the gravitational attraction between relatively small objects. His experiment was so well executed that it was not for a hundred years before a more accurate measurement was made [1].

My attempt to modernize the Cavendish experiment used a position sensitive detector (PSD) which, when combined with a signal processing circuit, can read out the position of the beam as a voltage. The PSD consists of a monolithic PIN photodiode that provides high levels of accuracy in its measurements. The measured data was then recorded using computer data acquisition and LabVIEW.

THEORY

In 1686, Isaac Newton published the *Principia Mathematica,* in which he stated the law of universal gravitation, written as

$$F = G\frac{m_1 m_2}{b^2}, \qquad (1)$$

where m_1 is the mass of one sphere, m_2 is the mass of the other sphere, b is the distance between them and G is the gravitational constant which Cavendish quantified in his experiments. The torsion balance consists of two small spheres at opposite ends of a thin rod. The rod is suspended from the center by the torsion wire. The gravitational attraction between the two small masses and the neighboring large masses produces a torque. This torque is equal and in the opposite direction of the torque provide by the twist of the torsion wire. Together they are expressed as

$$k\theta = \tau_{wire} = \tau_{grav} = 2dG\frac{m_1 m_2}{b^2}, \qquad (2)$$

where d is half the length of the pendulum arm, k is the torsion constant of the wire and θ is the angle through which it has been twisted. A schematic is shown in Figure 1.

Considering the fact that the measured angle between the incident beam and reflected beam is 2θ, θ is expressed as

$$\theta = \Delta S / 4L. \qquad (3)$$

Here ΔS is the change in the position of the reflected beam from the equilibrium point with no spheres and equilibrium point with the spheres at Position 1 or Position 2.

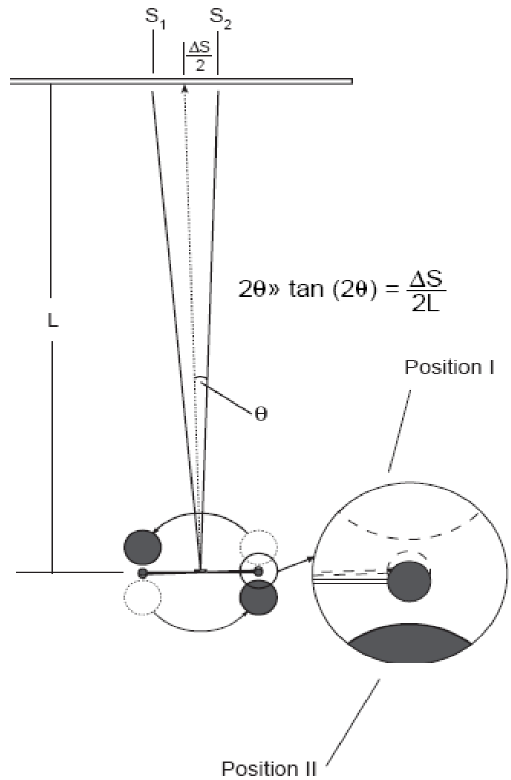

Figure 1: Diagram of reflected light beam at the two equilibrium points [1].

The torsion constant k, can be determined by observing the period T, of the oscillations and can be written as

$$k = 8\pi^2 m_2 \frac{\left(d^2 + \frac{2}{5}r^2\right)}{T^2}, \qquad (4)$$

where r is the radius of the small spheres. Substituting equations (3) and (4) into equation (2) gives

$$G = \pi^2 \Delta S b^2 \frac{\left(d^2 + \frac{2}{5}r^2\right)}{T^2 m_1 L d}. \qquad (5)$$

Because of the sensitivity of the apparatus, the gravitational force due to the second more distant large sphere had to be taken into account using a correction factor:

$$\beta = \frac{b^3}{\left(b^2 + 4d^2\right)^{3/2}}. \qquad (6)$$

The corrected gravitational constant can then be expressed as [1]

$$G_{corr} = \frac{G}{(1-\beta)}. \tag{7}$$

SET UP AND PROCEDURE

The torsion balance was placed on a vibration-dampening table to increase the accuracy of the experiment. The laser and PSD were positioned on a specially built mount so that the laser reflected from the mirror on the pendulum arm and hit the sensor. The PSD was connected to a data acquisition device which, through the use of a computer and a LabVIEW program, collected the data.

To perform the experiment the large spheres were placed in Position 1 and after an equilibrium point was reached, moved to Position 2.

ANALYSIS AND RESULTS

The torsion balance behaves as a damped simple harmonic oscillator. The data collected was fit using a damped sinusoidal function with an exponential decay, specifically,

$$f(t) = A\exp\left[-\frac{(t-t_0)}{\tau}\right]\sin(\omega(t-t_0)+\varphi)+y_0$$

where A is the amplitude, t_0 is the start time, $1/\tau$ is the decay rate, ω is the angular frequency, φ is the phase shift and y_0 is the offset or equilibrium point. Table 1 and Table 2 contain the values of the fit to Position 1 and Position 2 respectively.

Table 1: Fit values for Position 1.

	Value	Error	Units
A	2.2	± 46.7	V
t_0	5374	--	s
$1/\tau$	0.000912	± 0.000002	1/s
ω	0.012278	± 0.000002	rad/s
φ	-4	± 284	rad
y_0	0.00005	± 0.00054	V

Table 2: Fit values for Position 2.

	Value	Error	Units
A	-2.1	± 0.03	V
t_0	0	--	s
$1/\tau$	0.000869	± 0.000003	1/s
ω	0.012269	± 0.000003	rad/s
φ	-1.3	± 2.2	rad
y_0	1.8071	± 0.0008	V

Figure 2 shows the plot of the data collected as well as the fits made for each sphere position. From the plot, it is easy to see that the equation used to fit the data does a good job representing the data.

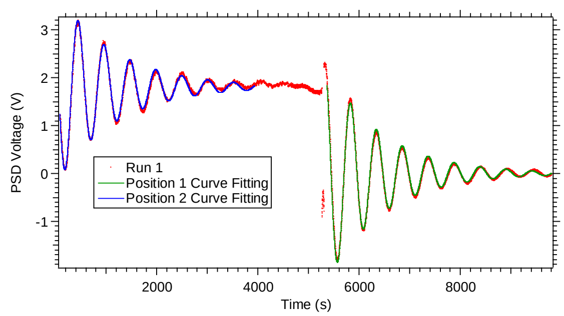

Figure 2: Plot of data run and curve fits to the different sphere positions.

From the fit, ω is used to find the period of oscillation using the expression [2]

$$T = \frac{2\pi}{\omega}. \tag{8}$$

The value I used for my calculations is $\omega = 0.01233 \pm 0.00008$. The value obtained for T is shown in Table 3.

The equation

$$x = \frac{37(VO)}{20} \text{ (mm)} , \qquad (9)$$

can be used to convert the voltages outputted by the PSD into distances from the center of the PSD, where *x* is the distance between the center of the PSD and the equilibrium position [3]. Using y_0 for both Position 1 and Position 2 the difference ΔS, between the two equilibrium points S_1 and S_2 can be calculated, also as shown in Table 3.

Table 3 also shows the values for *b, d,* and *r,* which were measured with calipers, and *L* and m_1, measured with a ruler and a electronic scale, respectively.

Table 3: Measurements for the variables *T,* ΔS, *b, d, r, L* and m_1.

	Value	Error	Units
T	509	± 3	s
ΔS	3.34304×10^{-3}	$\pm\ 5\times10^{-9}$	m
b	0.045	$\pm\ 5\times10^{-4}$	m
d	0.050	$\pm\ 5\times10^{-4}$	m
r	0.0075	$\pm\ 5\times10^{-5}$	m
L	0.147	$\pm\ 5\times10^{-4}$	m
m_1	1.49863	$\pm\ 5\times10^{-6}$	kg

Using equation (5) and the values above, I calculated $G = 5.89\times10^{-11}\ \text{Nm}^2/\text{kg}^2$.

With equation (6) I found the correction factor to be $\beta = 0.069 \pm 0.003$ which gave $G_{corr} = 6.33\times10^{-11}\ \text{Nm}^2/\text{kg}^2$.

The uncertainty of the corrected gravitational constant was calculated to be $\delta G_{corr} = 2.5\times10^{-12}\ \text{Nm}^2/\text{kg}^2$. Compared to $G_A = 6.67\times10^{-12}\ \text{Nm}^2/\text{kg}^2$ [4], the accepted value, gave a percent difference of 5.1%.

CONCLUSION

By measuring the amount of deflection caused by the attraction of two sets of two spheres using a laser and a mirror, one can determine the gravitational constant. Following the same basic procedure as Henry Cavendish used in his experiment in 1798, I was able to calculate the gravitational constant to a value of $6.3\pm0.3\times10^{-11}\ \text{Nm}^2/\text{kg}^2$ which only has a 5.1% difference from the accepted value. This error could be due to the measurements of *r* and *L* since they were done using rather basic techniques.

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