

# Frustrated Total Internal Reflection

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This experiment tests the theory for frustrated total internal reflection using light in the visible spectrum. A decaying exponential relationship between the intensity of a transmitted light beam and the distance between two media was found. The results are relevant for an undergraduate optics physics course or quantum mechanics course where it is analogous to barrier penetration.

## INTRODUCTION

J. C. Bose was examining the wave nature of the radiation of microwaves in 1897. His experiment consisted of a beam of microwaves directed at a right angle asphalt prism. The microwaves were totally internally reflected by the hypotenuse of the prism. However, when a second right angle prism was placed in contact with the first, hypotenuse to hypotenuse, the beam of microwaves passed through. The distance between the two prisms was increased, but kept significantly smaller than the wavelength of the microwave. A portion of the beam was transmitted through the prisms and across the gap between them. This confirmed the wave nature of microwaves.<sup>1</sup>

In order for this to occur, the wave passing through the first prism must have penetrated into the air gap between the prisms. When the gap is small enough, the wave is able to pass through the second prism as well. The wave that is penetrating the barrier of air between the two prisms is called the evanescent wave. Carniglia and Mandel found that evanescent waves are not different from homogeneous waves, when the photoelectric emission of a bound charge is under the influence of an evanescent wave.<sup>2</sup>

Hall studied the experimental and theoretical ideas of the transmitted wave and published it in 1902.<sup>3</sup> His method consisted of introducing a third material identical to the first (the prisms in Bose's experiment) and placing it very close to the first, creating a thin area between the identical materials. The thickness of the material between the two identical materials must be on the order of the wavelength of the wave used. The total reflection of light is frustrated. The materials used are assumed to be transparent. Hall studied the distance that the wave penetrated into the barrier with respect to the angle of incidence and the polarization of the incident radiation beam for several different materials. He

found that the intensity of the transmitted light increases as the distance the wave penetrates into the barrier region increases.<sup>3</sup> He also observed that the penetrated distance increases as the indices of refraction of the two media decrease.<sup>3</sup>

The transmission coefficient in terms of the distance between the two identical materials was found using Maxwell's equations. They were verified using centimeter wave radiation and the experiment first used by Bose. Hall's theoretical work was extended by Eichenwald, Foersterling, and Arzelies. They examined the energy flow, and found that the evanescent wave decayed exponentially in the barrier.<sup>4</sup> With the development of quantum mechanics, barrier penetration was found to be analogous to the frustrated total internal reflection of optics.

## THEORY

A beam of light that is incident on a reflective surface at an angle  $\theta_i$  will be reflected at an angle  $\theta_r$ , according to the Law of Reflection:  $\theta_i = \theta_r$ . The angles are measured from the normal to the surface. Both the incident and reflected beams of light lie in one plane, the plane of incidence. However, if a beam of light is incident on a surface that is not completely reflective, the beam will "bend" as it crosses the boundary. The light does not actually bend, but its speed changes, resulting in the transmitted light traveling at a different angle, the transmittance angle  $\theta_t$ . The medium that the plane of incidence lies in has the index of refraction of  $n_i$ , and the medium the transmitted plane is in has the index of refraction of  $n_t$ . The angle of incidence and the transmitted angle relate to each other by their respective indices of refraction according to Snell's Law,  $n_i \sin(\theta_i) = n_t \sin(\theta_t)$ .

In the case of internal reflection (where  $n_t > n_i$ ) all the incoming light is reflected back into the incident medium when the incident angle is

greater than or equal to the critical angle,  $\theta_c$  (which is the incident angle for which the transmitted angle is equal to 90 degrees).

While it does not appear that there is a transmitted wave, it does exist, it just cannot carry energy across the boundary. The intensity,  $I$ , of the transmitted light is given by<sup>5</sup>

$$I \propto e^{-\alpha \frac{D}{\lambda_0}}, \quad (1)$$

where

$$\alpha = 4\pi n_i \left[ \frac{n_i^2}{n_t^2} \sin^2 \theta_i - 1 \right]^{1/2}, \quad (2)$$

$D$  is the distance between the two media, and  $\lambda_0$  is the wavelength of light in a vacuum.

So, if the angle of incidence and the indices of refraction of the two media are known, the graph of the intensity of the transmitted light beam versus the ratio of the distance between two media to the wavelength of light would follow an exponential decay. Equation 2 can be used to find the value of  $\alpha$  which can be compared to a calculated value from an exponential fit of a graph of the intensity of the transmitted light beam versus the ratio of the distance between the two media to the wavelength of light.

## EXPERIMENT

The method used consisted of two flat glass Fabry-Perot mirrors ( $n_{\text{glass}}=1.51509$ ) secured into holders on a Hilger & Watts translation stage with their coated sides facing each other. The glass flats were cleaned thoroughly using lens paper so that there were no debris on the flats. To make sure the flats were parallel to each other, a Melles Griot HeNe laser was used to roughly align them, followed by a Hydrogen-Deuterium source placed at the focal point of a lens. A filter was placed in between the lens and the glass flats, which were aligned so circular fringes were observed. As the flats were moved closer together, the interference fringes (which look like concentric rings getting smaller then disappearing, while other rings appear on the outside) occur. At the point where the fringes stop, the distance between the flats is approximately less than one half the wavelength of light.

A small, right angle prism was attached to the glass disk in the movable translation holder using decahydronaphthalene, a liquid material that has a similar index of reflection as the prism and the glass. The decahydronaphthalene was added drop by drop using a toothpick to the hypotenuse of the prism and then attached by the surface tension of the liquid. The prism and glass flat will then act as a single material with the same index of reflection, so when the light beam passes from

the prism to the glass flat through the liquid material, it will not deflect. In order for the transmitted light beam to exit the glass flat and be observed, a second prism needed to be attached to the glass flat in the stationary holder. See Figure 1. The second prism was the exact same as the first prism, and attached to the glass disk in the same manner.

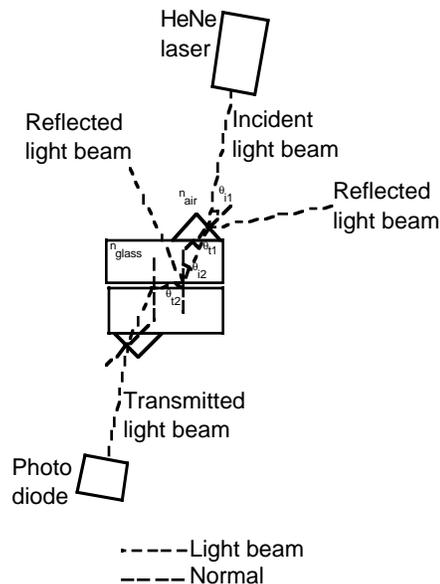


FIG. 1. This was the arrangement of glass flats, prisms, HeNe laser, and photo diode. Some of the light from the HeNe laser is reflected off of the surface of the prism while the rest of it goes into the prism and glass flats. At the interface between the glass flat and the air of the gap, some light is reflected also.

The Melles Griot HeNe laser ( $\lambda=633$  nm) was moved more perpendicular to the glass disks, and at an angle of approximately 10 degrees from the normal of the first prism. This resulted in the incident angle (incident on the interface from glass to the air in the gap between the two glass flats) of the light beam inside the glass to be greater than the critical angle. This makes sense because in order for total internal reflection to occur, the angle of incidence must be greater than or equal to the critical angle.

A photo diode connected to a United Detector Technology optometer was used to measure the intensity of the transmitted light beam. The photo diode was positioned so that its screen was perpendicular to the transmitted light beam, making sure that the whole beam was hitting the screen.

The intensity of the transmitted light beam at different distances between the glass flats was measured, starting when the glass flats were touching. Because Fabry-Perot flats were used, the intensity of the light increased, then decreased, then increased again, then decreased again, and so on as the glass flats were moved apart. The

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reason for the interference fringes, and therefore the variation in the intensity of the transmitted light beam, was the coating on the glass flats. The coating is partially reflective, so light bounces in between the glass flats, only some of the light getting out each time. In order to compensate for this while taking data, intensity measurements were only taken at the peak intensities, when the beam was at its most intense. While looking at the screen of the photo diode, the distance between the glass flats was increased very slowly until the intensity of the beam was at its greatest. At this time, the maximum intensity reading and the distance between the glass flats were recorded. Then, the distance was again increased very slowly. The intensity of the next maximum fringe and the distance between the flats were recorded. The intensity for the fringes and the distances between the flats were recorded in this manner until there was little change in the maximum intensity of subsequent readings.

### DATA AND DISCUSSION

The value for the angle of incidence in the first interface was found using geometry. A meter stick was placed a measured distance away from the first prism, but not as far away as the HeNe laser. The distance between where the light beam entered the prism and the zero point of the meter stick was the measured distance the meter stick was away from the prism. Knowing these distances and the distance between the meter stick and the prism, the angle of incidence was calculated to be  $9.53^\circ$ .

The procedure that was used to take the data necessitated the advancing of the distance between the glass disks at a very slow speed so the advancing could be stopped at the maximum intensity of the light beam hitting screen of the photo diode. Since the maximum comes and goes so quickly, the instant that the maximum occurred, the intensity reading needed to be taken. Instead of watching the photo diode screen to see when the maximum intensity occurred, the intensity meter was watched. This allowed the maximum intensity of the light beam reading to be made for each of the intensity fringes. The second set of data taken verified that this process worked because there were no outlying intensity measurements.

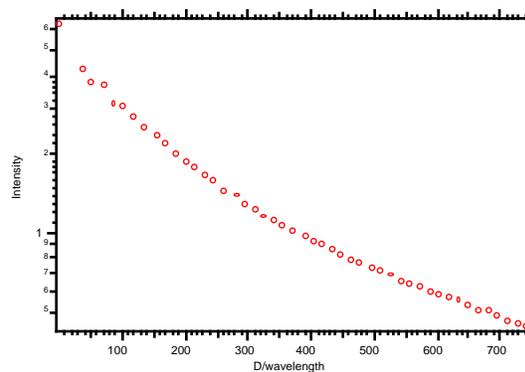


FIG. 2. This is the graph of the second set of

The graph in Figure 2 is the intensity of the transmitted light beam plotted on a log scale on the y-axis and the ratio of the distance,  $D$ , between the glass flats to the wavelength of light. The general trend for the graph appears to be a double exponential. If it was a single exponential, then the data would form a linear pattern. There appears to be two different linear patterns for this graph, one ranging from where  $D/\text{wavelength}$  is zero to approximately 150 or so. The other linear pattern ranges from where  $D/\text{wavelength}$  is about 150 through 750.

It is speculated that one exponential function describes the reflective coating of the Fabry-Perot glass flats while the other describes the intensity of the light beam as a function of the ratio of  $D$  divided by the wavelength of light.

Igor Pro 4.01 was unable to fit a satisfactory double exponential curve to this data on its own. So, one of the parameters of the function needed to be set for it. Since one of the exponential functions describes the intensity of light as opposed to the reflective properties of the glass flats' coating, Equation 2 was used to find the value of  $\alpha$ . The value of alpha then relates the intensity of the light beam to the ratio of  $D$  divided by the wavelength of light through Equation 1.

The transmitted angle for the first interface (between air and the first prism) was found using Snell's Law, with the angle of incidence and the indices of refraction as stated earlier. Using this angle, the angle of incidence for the second interface (between the first glass flat and the air of the gap) was found to be  $51.3^\circ$ . This angle is necessarily greater than the critical angle, which is  $41.3^\circ$ .

### RESULTS

The value for the variable  $\alpha$  in Equation 1 was found using Equation 2 with the angle of incidence and the indices of refraction for the second interface (between the first glass flat and the air of the gap) stated earlier. The calculated

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value of  $\alpha$  ( $\alpha = 7.92$ ) allowed Igor Pro 4.01 to find a double exponential function that fit the graph of the intensity of the transmitted light beam versus the ratio of the distance between the glass flats to the wavelength of light .

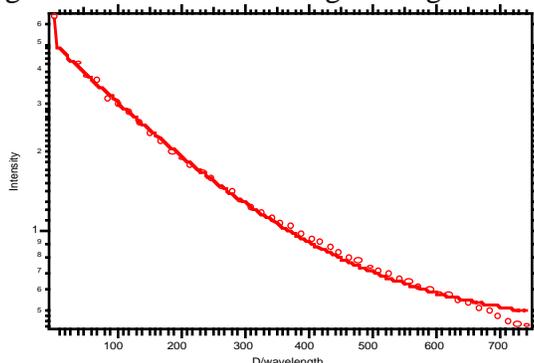


FIG. 3. This is a plot of the second set of data. The ratio of D to the wavelength of light is plotted on the x-axis and the intensity of the transmitted light beam is plotted on a log scale on the y-axis. A double exponential curve was fit to the data:

$$I = 9.57 \times 10^{10} e^{-7.92 \frac{D}{\lambda_0}} + 4.64 e^{-0.006 \frac{D}{\lambda_0}} + 0.431 .$$

### CONCLUSION

This experiment yielded sufficient data that supported the theory for frustrated total internal reflection using visible light. Good quantitative results were not obtained, but a decaying exponential relationship between the intensity of a transmitted light beam and the distance between two media was found. In order to yield accurate quantitative results, the distance between the two glass flats needed to be measurable at distances less than one wavelength of light. The Hilger & Watts translation stage used only had resolution to one micrometer instead of ten nanometers (which would be a sufficient division of the wavelength of light). The translation stage was, however, able enough to yield a good qualitative picture. Another factor that halted a deeper exploration into the transmission coefficients of the tunneled light beam was the crude accuracy of the optometer, which measured the intensity of light. Quantitative results may also be able to be obtained with glass flats that did not have any reflective coating on their surfaces.

<sup>1</sup>Ghose, Partha, *Testing Quantum Mechanics on New Ground*. Cambridge University Press: Cambridge, England, 1999, p 29.

<sup>2</sup>Carniglia, C.K. and L. Mandel, "Quantization of evanescent electromagnetic waves," *Physical Review D*. 3 (1), 280-296 (1971).

<sup>3</sup>Hall, E. E., "The penetration of totally reflected light into the rarer medium." *Physical Review*. 15, 73-106 (1902).

<sup>4</sup>Zhu, S., A. W. Yu, D. Hawley, and R. Roy, "Frustrated total internal reflection: A demonstration and review," *American Journal of Physics*. 54 (7), 601-606 (1986).

<sup>5</sup>Hecht, Eugene, *Optics*, 4<sup>th</sup> ed. Addison Wesley: New York, 2002, p 125.